Discounting the Distant Future: How Much Do Uncertain Rates Increase Valuations?

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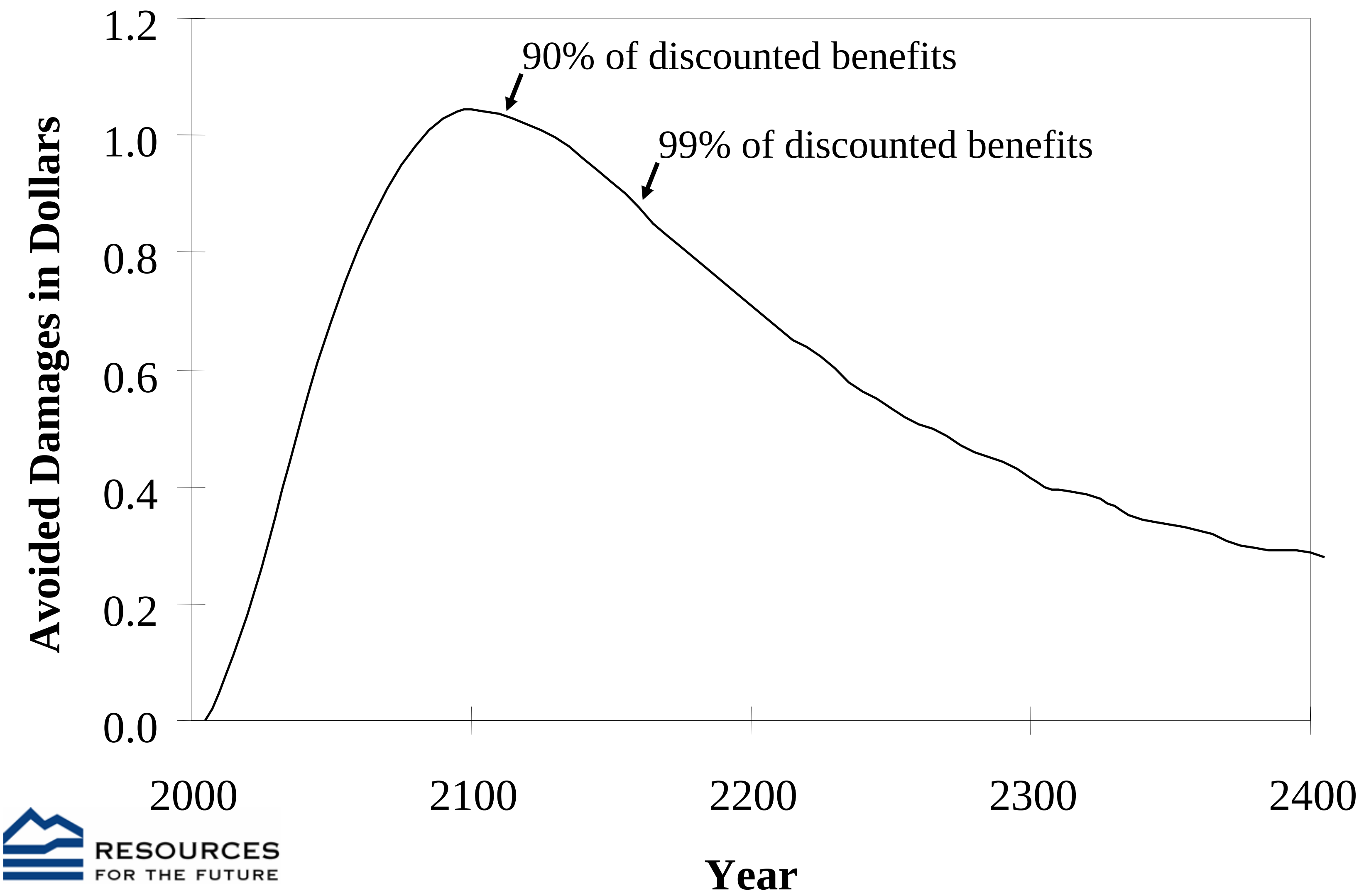
Motivation

- Standard geometric discounting renders values in the distant future worthless
  - $1 delivered in 200 years is worth 6/1000¢ today at a 5% discount rate
- Huge effect on climate policy analysis where benefits occurs hundreds of years in future (also: long-lived infrastructure, toxic/radioactive waste, biodiversity)
- "Seems wrong." (Ainslie; Cropper, Aydede, & Portney; Bazerlon & Smetters). But, suggested modifications to standard framework suffer from time inconsistency.
- Work by Weitzman (1998, 2001).

Future Consequences of 1 ton of Carbon Mitigation in 2000



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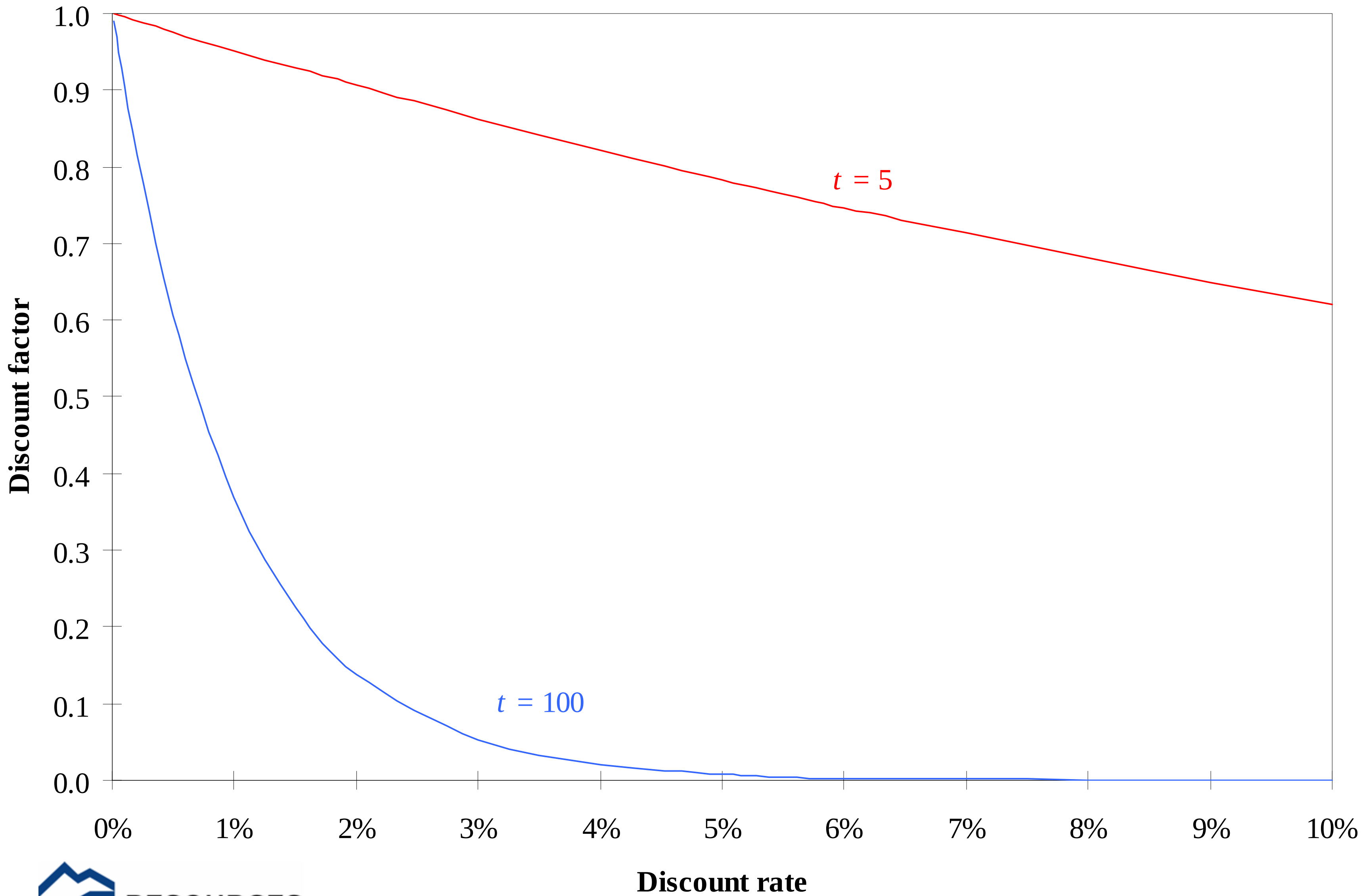
Weitzman Point



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Weitzman Point

- Equal likelihood of rate r_1 = 0% or r_2 = 10%
- Expected discount factor: $\mathrm{E}[P_t] = \frac{1}{2}\left(e^{-r_1 t} + e^{-r_2 t}\right)$
- Certainty-equivalent rate (rate of change in discount factor):

$$\tilde{r}_t = -\frac{d\mathrm{E}[P_t]/dt}{\mathrm{E}[P_t]} = \underbrace{r_1\left(\frac{e^{-r_1 t}}{e^{-r_1 t} + e^{-r_2 t}}\right)}_{\text{weight on } r_1} + \underbrace{r_2\left(\frac{e^{-r_2 t}}{e^{-r_1 t} + e^{-r_2 t}}\right)}_{\text{weight on } r_2}$$

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Our approach

- Geometric discounting using market-revealed rates
- Current rate observed accurately but future discount rate is *uncertain*
- Leads to an increase in expected discounted values and a decline over time in certainty-equivalent rates (theory)
- Measure future discount rate uncertainty based on time-series analysis of 200 years of interest rates (empirical)
- Forecast certainty-equivalent discount rate path based on *alternate* assumptions about the discount rate
- Determine appropriate discount factors

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Stochastic model of interest rates

- Autoregressive model of interest rate *r*

$$r_t = \eta + \varepsilon_t \qquad \varepsilon_t = \rho \cdot \varepsilon_{t-1} + \xi_t$$

$$\eta \sim \mathcal{N}\left(\bar{\eta}, \sigma_\eta^2\right) \qquad \xi \sim \mathcal{N}\left(0, \sigma_\xi^2\right)$$

$\bar{\eta}$ = mean interest rate

ρ = persistence (autocorrelation)

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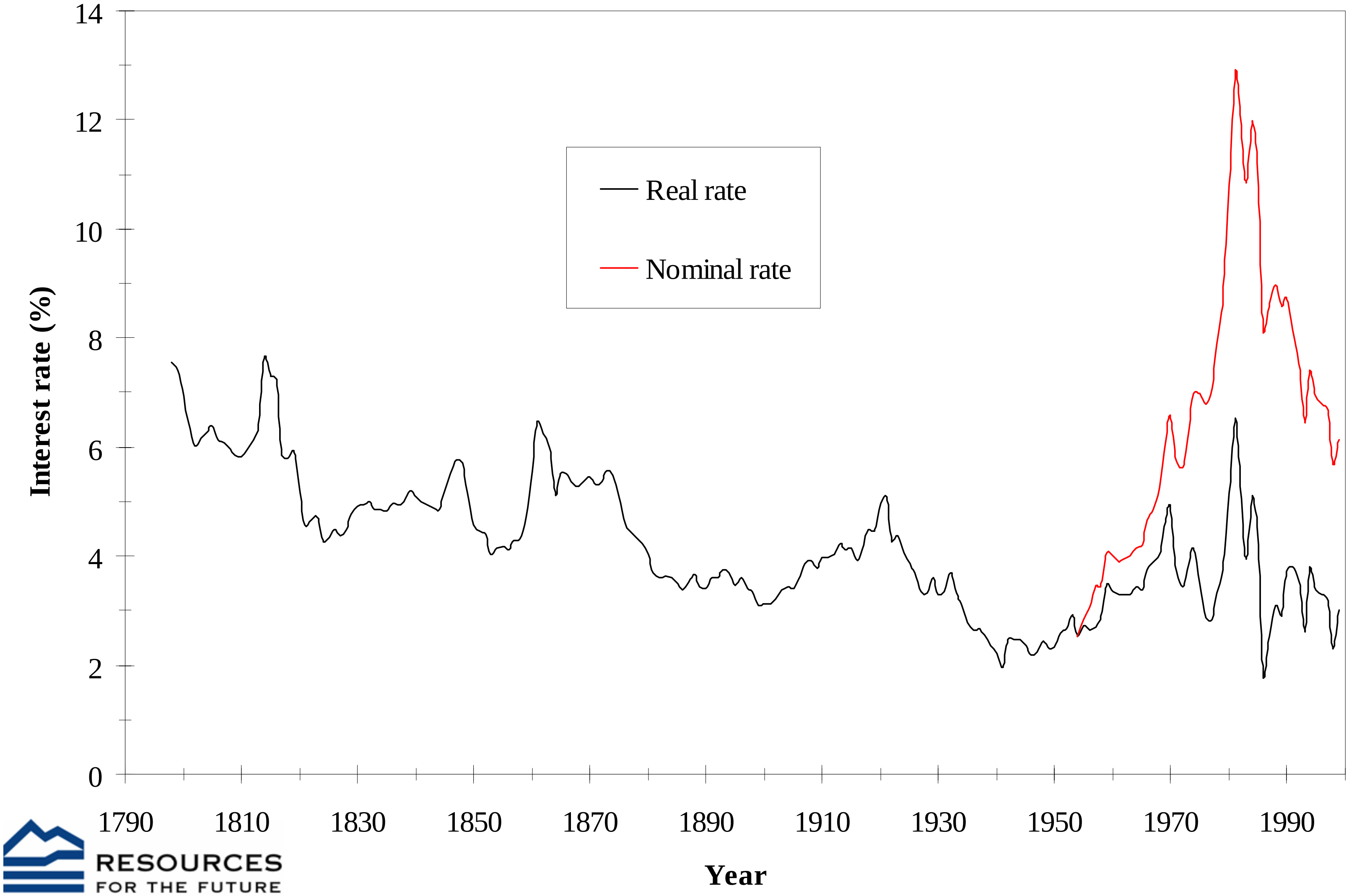
Market interest rate for U.S. long-term government bonds (1798-1999)



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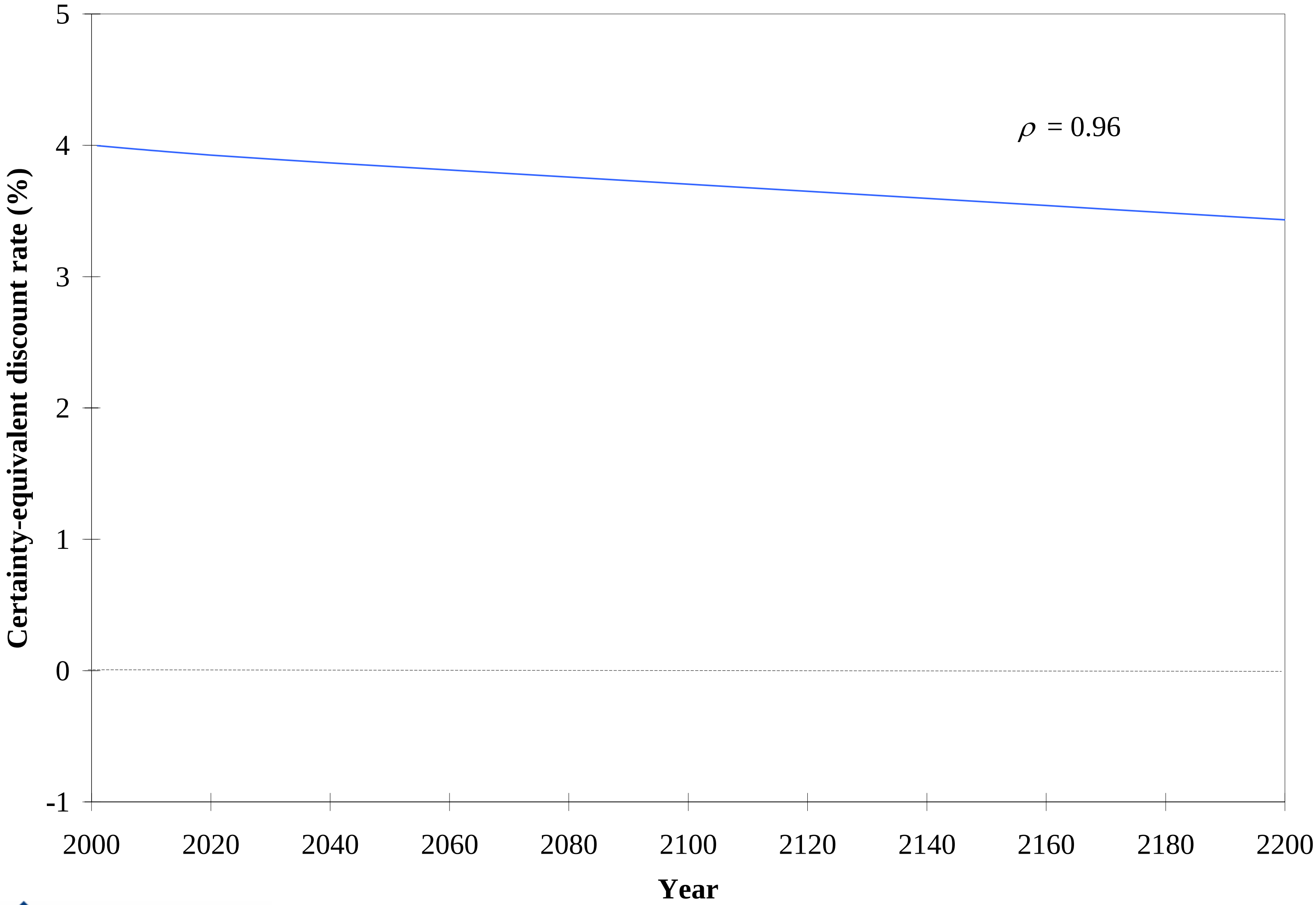
Importance of persistence



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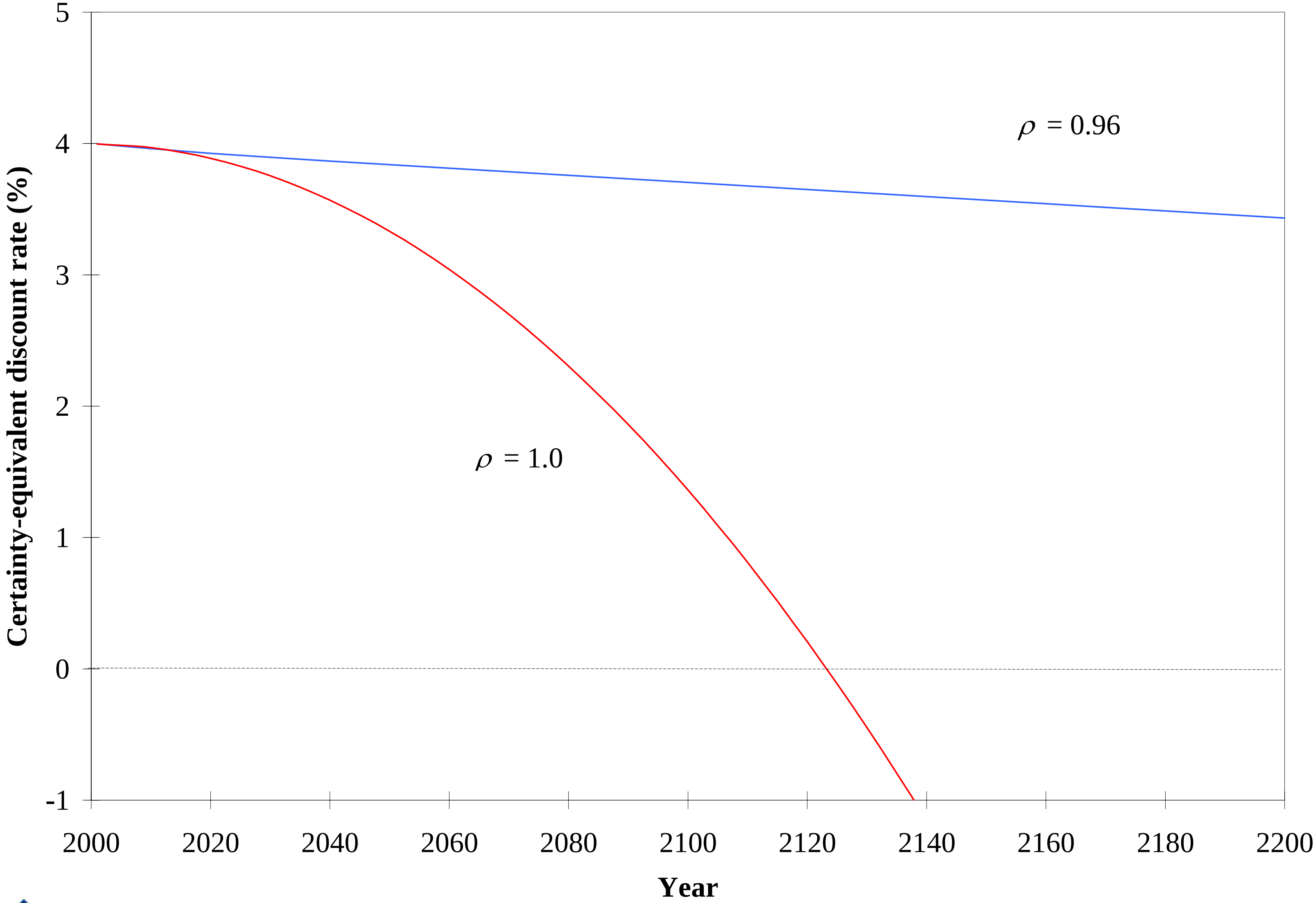
$$\sigma_{\xi} = 0.23\%, \bar{\eta} = 4\%, \sigma_{\eta} = 0.52\%, \rho = 0.96$$

Importance of persistence



$$\sigma_{\xi} = 0.23\%, \bar{\eta} = 4\%, \sigma_{\eta} = 0.52\%, \rho = 0.96 \text{ vs. } 1.0$$

Estimation of interest rate uncertainty

- Modifications to address issues with simple model
  - estimate in logs (disallow negative rates)
  - simulate over ρ uncertainty
  - Plus: allow more general autocorrelation (more lags)
- Cannot reject hypothesis that $\rho = 1$: two models
  - random walk model ($\rho = 1$: use differences)
  - mean-reverting model ($\rho < 1$: don't difference, treat ρ as random and reject draws > 1)
- Conditional maximum likelihood; Schwarz-Bayes information criterion to choose number of lags

Estimation results

(std errors in parentheses)

Table 1
Estimation results ($\ln r_t = \ln \eta + \varepsilon_t$ and $\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \cdots + \rho_L \varepsilon_{t-L} + \xi_t$)

	Random walk model ($\sum \rho_s = 1$)		Mean-reverting model		Simple unlogged model[a]	
Mean rate ($\bar{\eta}$)			3.69*,b	3.95*,b	3.52*	3.92*
Std error (σ_η)			0.45	0.23	0.52	0.31
Autoregressive coefficients[c]						
ρ_1	1.92*	1.92*	1.88*	1.85*	0.96*	0.94*
	(0.06)	(0.06)	(0.07)	(0.07)	(0.01)	(0.02)
ρ_2	−1.34*	−1.34*	−1.31*	−1.26*		
	(0.12)	(0.12)	(0.12)	(0.12)		
ρ_3	0.43*	0.43*	0.40*	0.36*		
	(0.07)	(0.07)	(0.07)	(0.07)		
Trend		−0.0037[d]		−0.0033*,d		−0.010[d]
		(0.0055)		(0.0010)		(0.005)
σ_ξ^2	0.0015*	0.0015*	0.0015*	0.0015*	0.0522*	0.0517*
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0052)	(0.0052)

*Significant at the 5% level.

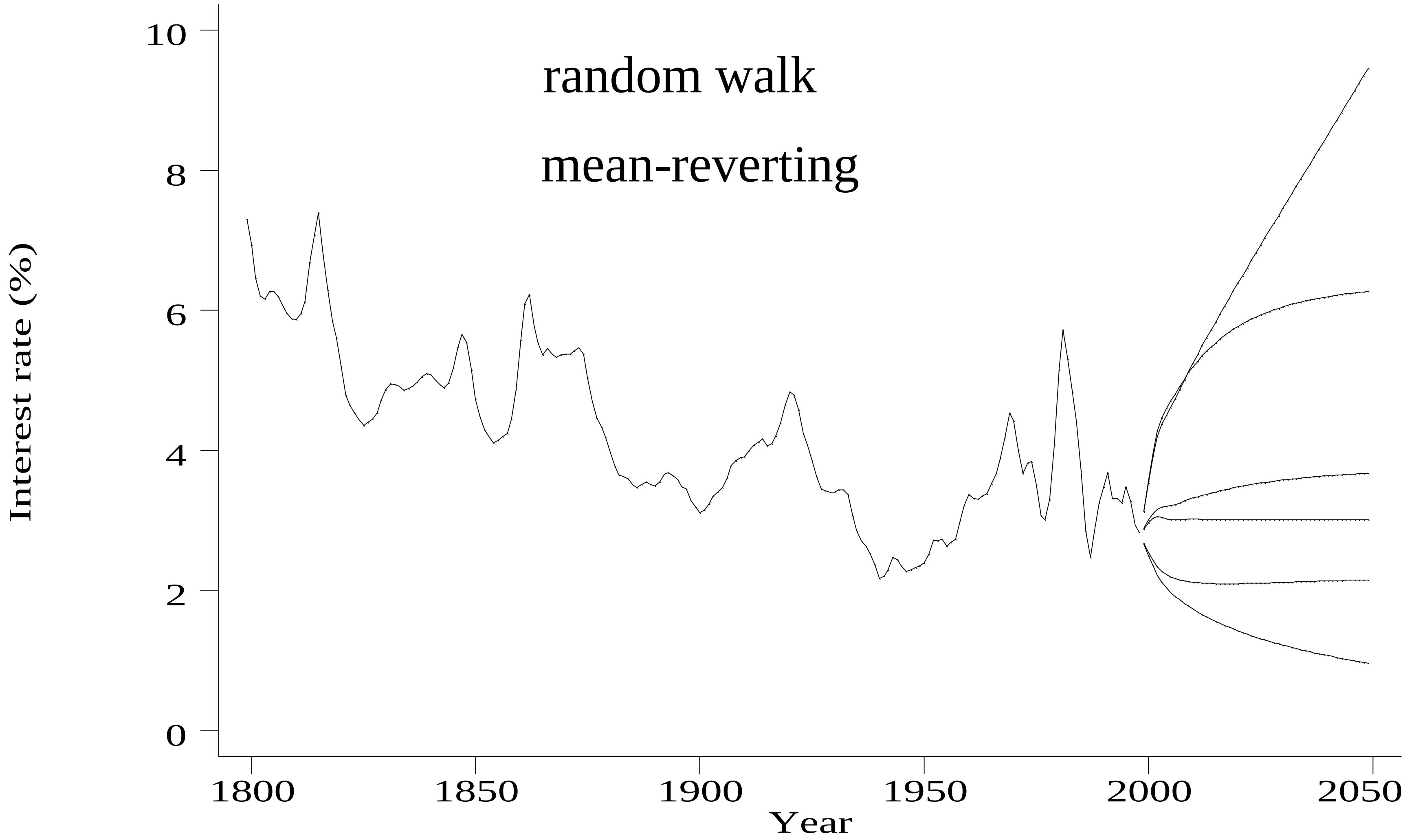
[a] Simple unlogged model is $r_t = \eta + \varepsilon_t$, where $\varepsilon_t = \rho\varepsilon_{t-1} + \xi_t$.

[b] The mean rates in this table were constructed, for simulation purposes, to reflect a continuously compounded rate. To convert to simple annual rates, simply compute $100 \times (\exp(\eta/100) - 1)$, e.g., $100 \times (\exp(3.69/100) - 1) = 3.76$.

[c] Number of autoregressive terms chosen using the Schwarz–Bayes information criterion.

[d] Indicates a linear time trend estimated alongside ε_t in each model.

Confidence intervals on interest rate forecasts



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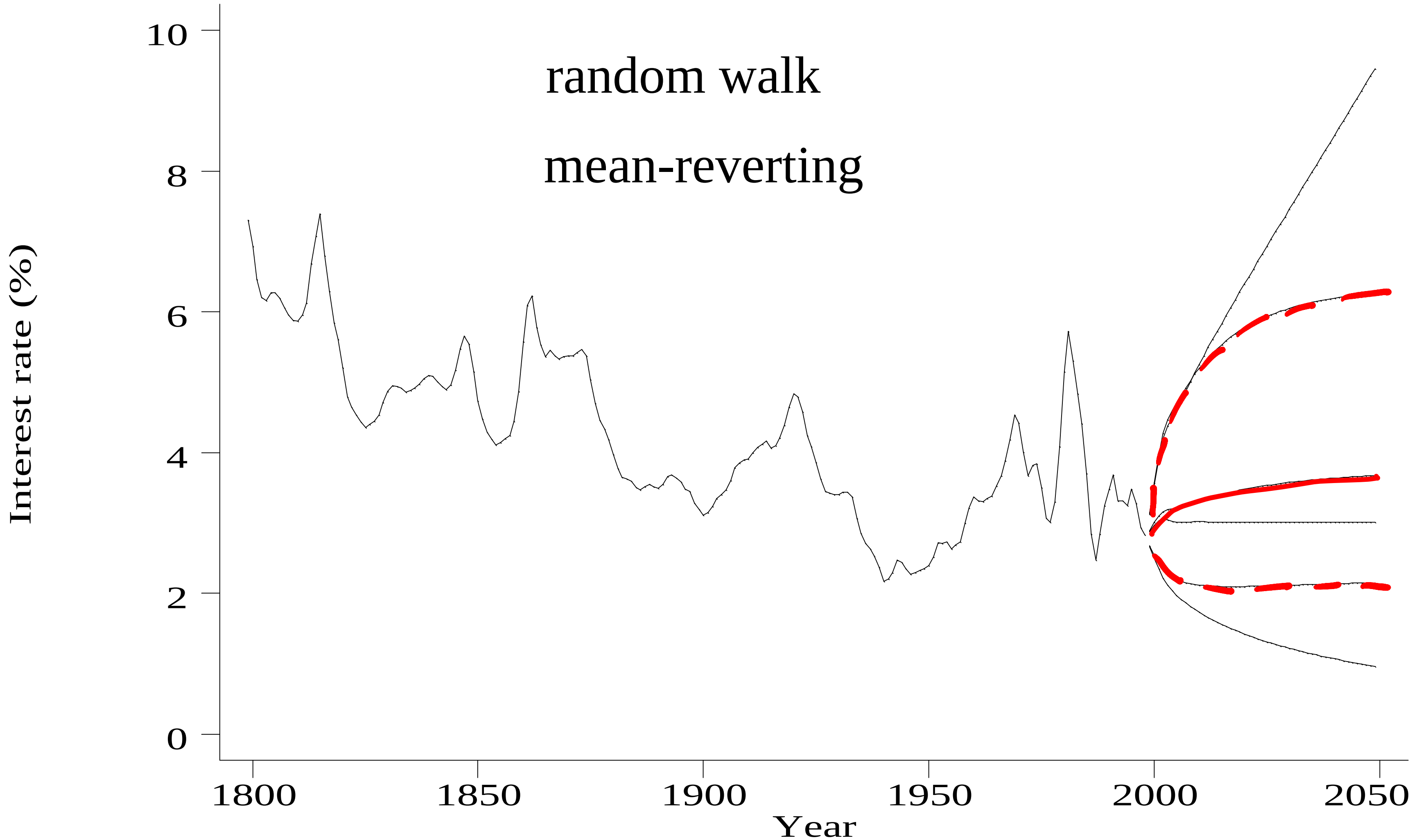

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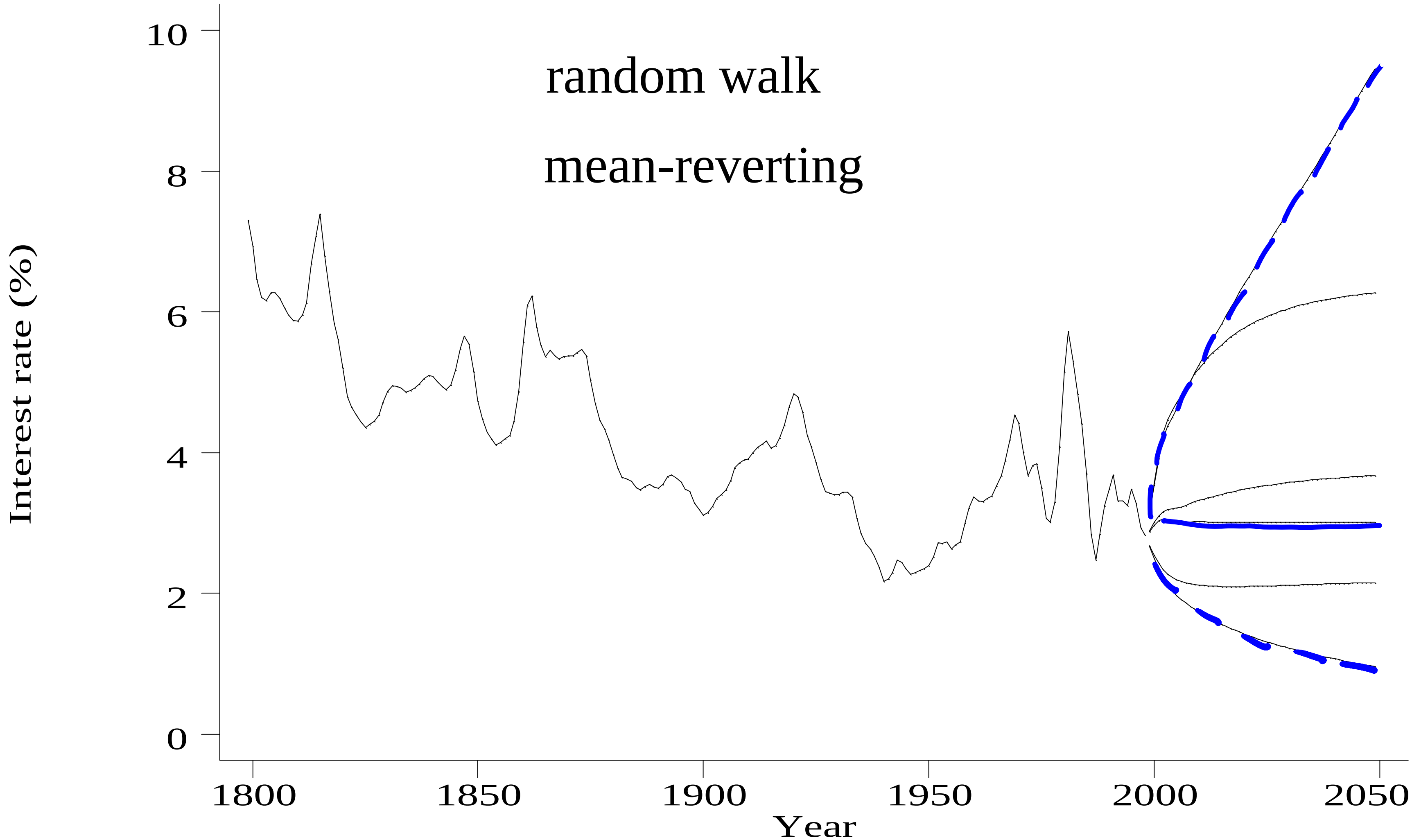

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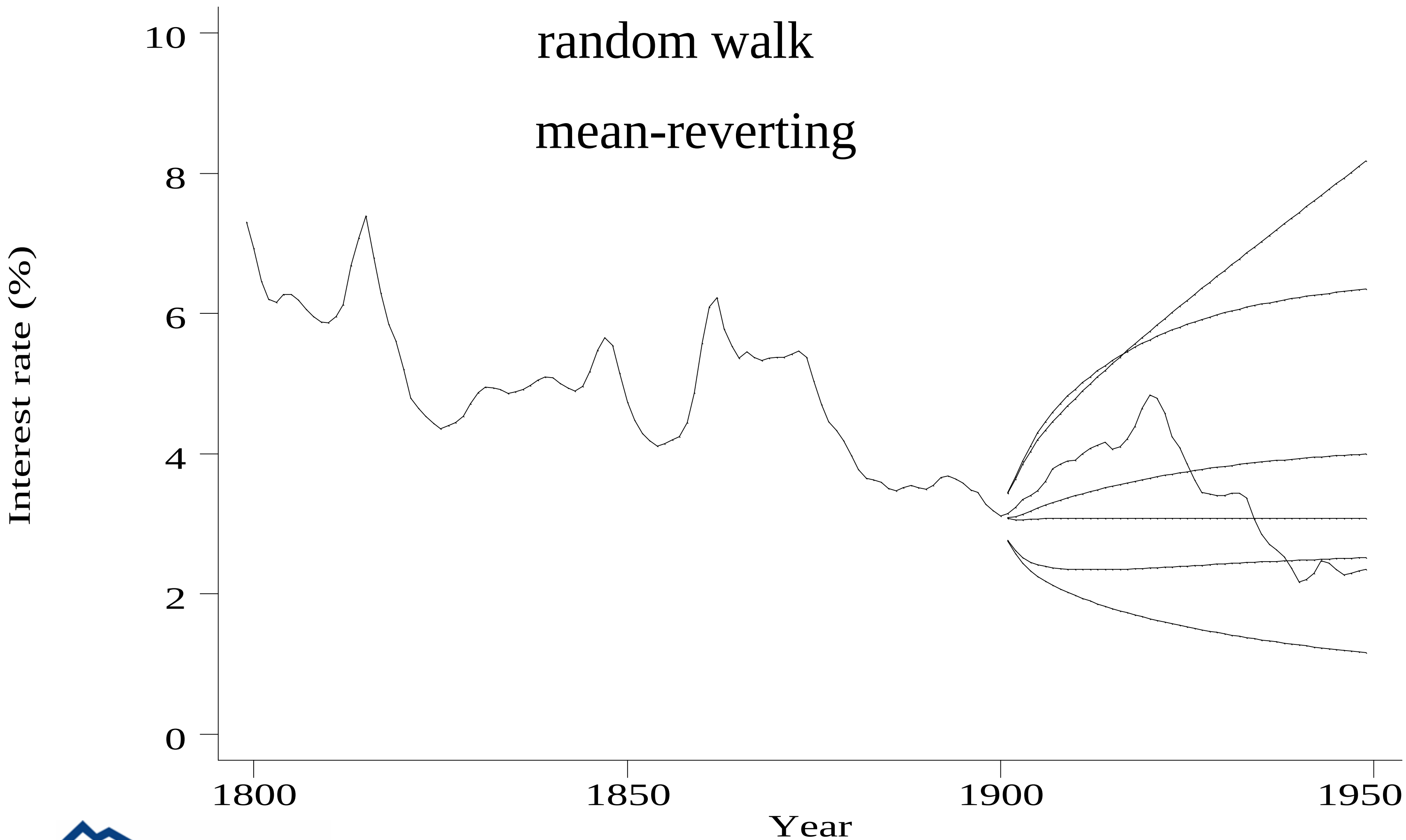

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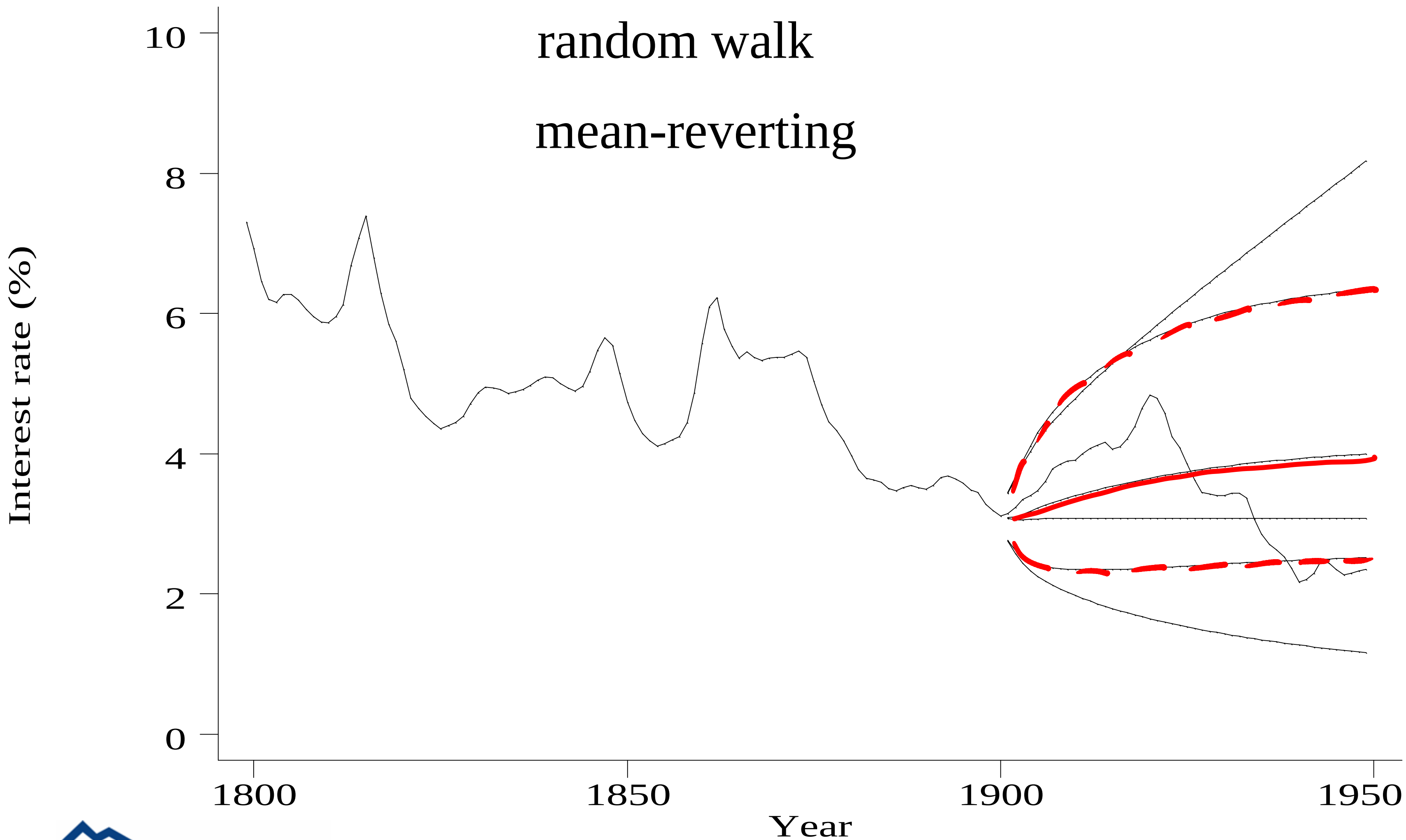

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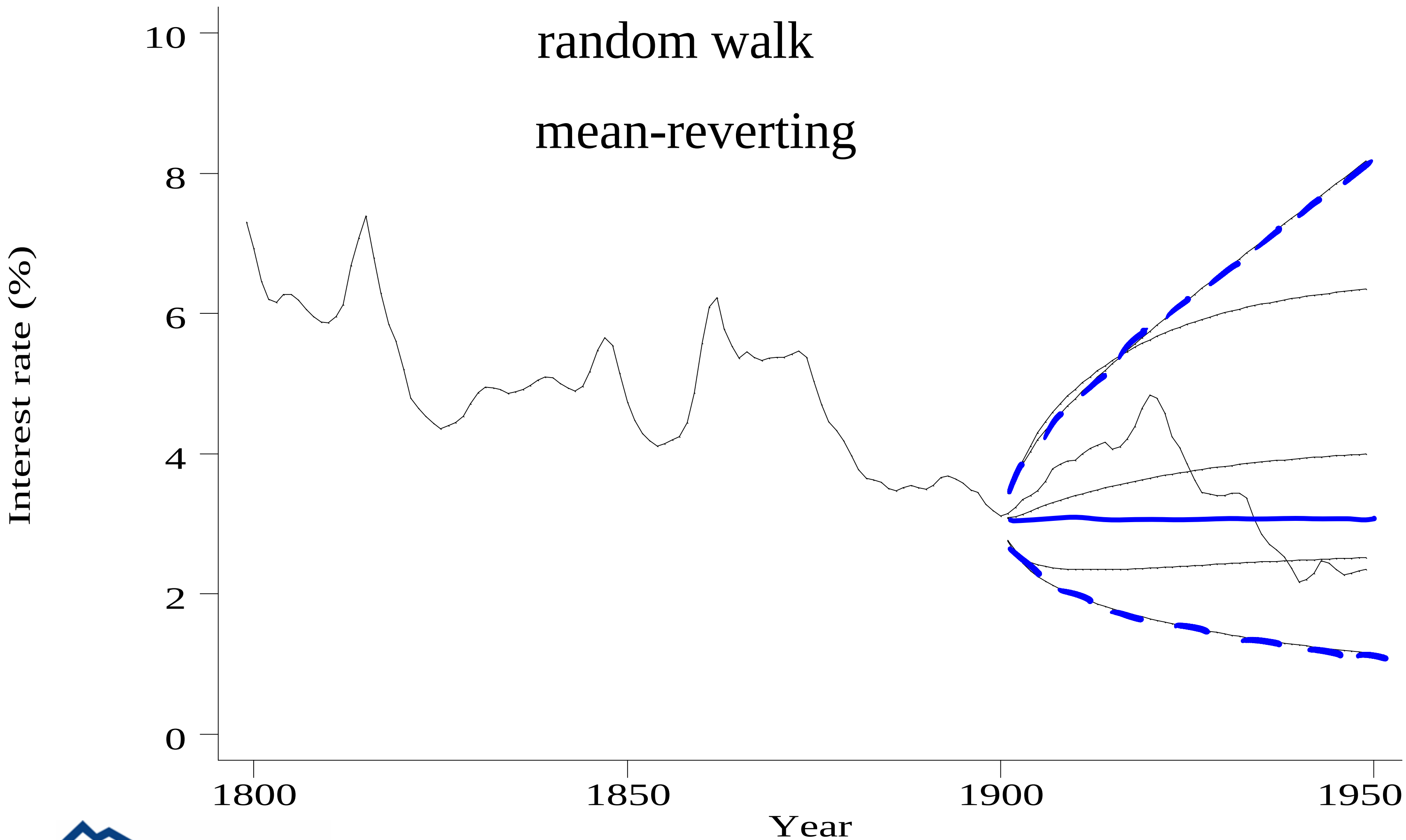

Simulations

- Draw parameters (ρ, η).
- Draw shocks (ξ).
- Construct disturbances (ε)
- Construct discount rates (r)
- Repeat
- Fix expected rate for different benchmarks.
- Construct expected discount factor: $\mathrm{E}\left[P_t\right] = \mathrm{E}\left[\exp\left(\sum_{s=1}^{t} r_s\right)\right]$
- Construct certainty equivalent rate:

$$\tilde{r}_t = \frac{d\mathrm{E}\left[P_t\right]/dt}{\mathrm{E}\left[P_t\right]}$$

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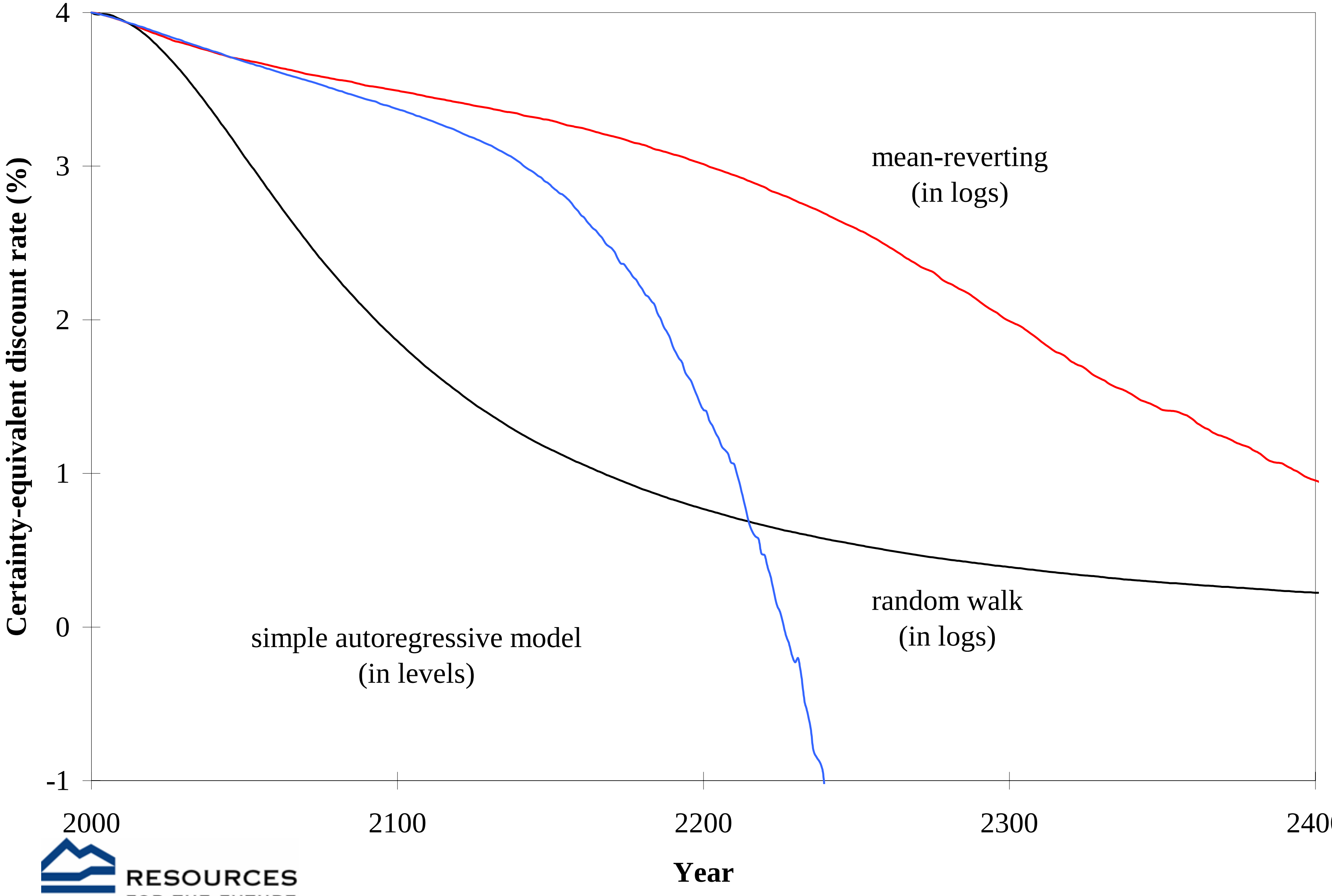
Forecasts of certainty-equivalent discount rates



Discount Factors: 4% rate

Years in future	Discount rate model			Value relative to constant discounting	
	Constant	Mean reverting	Random walk	Mean reverting	Random walk
0	$100.00	$100.00	$100.00	1	1
20	45.64	46.17	46.24	1	1
40	20.83	21.90	22.88	1	1
60	9.51	10.61	12.54	1	1
80	4.34	5.23	7.63	1	2
100	1.98	2.61	5.09	1	3
120	0.90	1.33	3.64	1	4
140	0.41	0.68	2.77	2	7
160	0.19	0.36	2.20	2	12
180	0.09	0.19	1.81	2	21
200	0.04	0.10	1.54	3	39
220	0.02	0.06	1.33	3	75
240	0.01	0.03	1.18	4	145
260	0.00	0.02	1.06	5	285
280	0.00	0.01	0.97	7	568
300	0.00	0.01	0.89	11	1,147
320	0.00	0.01	0.83	16	2,336
340	0.00	0.00	0.78	26	4,796
360	0.00	0.00	0.73	43	9,915
380	0.00	0.00	0.69	74	20,618
400	0.00	0.00	0.66	131	43,102

Discount Factors: alternate rates

ears in uture	2% initial rate			7% initial rate		
	random walk	constant rate	ratio	random walk	constant rate	ratio
0	100.00	100.00	1	100.00	100.00	1
20	67.54	67.30	1	26.89	25.84	1
40	46.48	45.29	1	8.67	6.68	1
60	33.05	30.48	1	3.52	1.73	2
80	24.42	20.51	1	1.75	0.45	4
100	18.76	13.80	1	1.02	0.12	9
120	14.93	9.29	2	0.67	0.03	22
140	12.25	6.25	2	0.47	0.01	62
160	10.32	4.21	2	0.36	0.00	181
180	8.89	2.83	3	0.29	0.00	557
200	7.81	1.91	4	0.24	0.00	1,778
220	6.97	1.28	5	0.20	0.00	5,851
240	6.30	0.86	7	0.17	0.00	19,726
260	5.77	0.58	10	0.16	0.00	67,829
280	5.33	0.39	14	0.14	0.00	236,788
300	4.97	0.26	19	0.13	0.00	837,153
320	4.66	0.18	26	0.12	0.00	2,992,921
340	4.40	0.12	37	0.11	0.00	10,804,932
360	4.18	0.08	52	0.10	0.00	39,298,213
380	3.99	0.05	74	0.10	0.00	143,866,569
400	3.83	0.04	105	0.09	0.00	529,656,724

Effect of discount rate uncertainty on discounted climate damages

		Benefits from 1 ton of carbon mitigation	Relative tc constant rat
Government bond rate (4%)	Constant 4% rate	$5.74	—
	Random walk model	$10.44	+82%
	Mean-reverting model	$6.52	+14%
2% rate	Constant 2% rate	$21.73	—
	Random walk model	$33.84	+56%
	Mean-reverting model	$23.32	+7%
7% rate	Constant 7% rate	$1.48	—
	Random walk model	$2.88	+95%
	Mean-reverting model	$1.79	+21%

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Summary of results

- Discount rate uncertainty implies a declining certainty-equivalent rate in the future
- Estimated uncertainty and persistence in long-term interest rates suggests the magnitude of this effect can be large
  - $\tilde{r}_t$ falls from 4% benchmark, to 2% after 100 years, to 1% after 200 years, to 0.5% after 300 years, based on random walk model
  - valuation 400 years in the future rises 43,000x
  - discounted climate damages almost double

EPA Questions

- How does N&P avoid time inconsistency?
- How do we apply N&P to Ramsey model?
- How do we choose the right benchmark rate? (consumption v. investment; pre- and post-tax)
- Are there other special characteristics of climate change investments that should be reflected in discount rates?

Time Consistency

- Problem: A decision is time inconsistent if we know now that we will want to change that decision in a certain way simply due to the passage of time.
- E.g., choose $1 in 2100 versus $1.03 in 2101.
- Compare *hyperbolic* discounting (4% now, 2% 100 years from now) versus *uncertain* discounting (4% now, 2% certainty equivalent 100 years from now).
- With uncertainty, the changing decision is a consequence of passage of time *and* new information.

Ramsey Discounting

- Ramsey model: maximize $\sum_t e^{-\rho t} U(C_t)$ subject to production function, capital accumulation
- Equilibrium condition: $r_t = \rho + \tau \cdot \left(\dot{C}_t / C_t \right)$ net return to capital (interest rate) equals pure time preference + growth discounting
- Choice of estimation: structural v. reduced-form
- Application to IAMs – why?

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Choice of Benchmark Rate

- Ethical Concerns
- Market Rates
  - 10%: Return to corporate investment.
  - 7%: Stock market yield.
  - 4%: Bond yield.
  - 2%: After-tax (personal income) bond yield.
- Consumption versus investment rate of interest; shadow price of capital approach.
- Risk-free rate.

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Climate Change and Discount Rates

- Typical approach is to separate out risk and discounting; discount risk-adjusted expectations at risk-free rate.
- Main concern is catastrophe: climate risk and interest rate are not uncorrelated in structural model

$$\sum_t r_t = t \cdot \rho + \tau \cdot \left(\Delta C_t / C_t \right)$$

- Otherwise, why treat climate change differently?

Thanks!

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Definitions

- Discount Factor $P_t = \exp\left(-\sum_{s=1}^{t} r_s\right)$
- Expected Discount Factor $\mathrm{E}\left[P_t\right] = \mathrm{E}\left[\exp\left(-\sum_{s=1}^{t} r_s\right)\right]$
- Certainty Equivalent Discount Rate

$$\tilde{r}_t = -\frac{d\mathrm{E}\left[P_t\right]/dt}{\mathrm{E}\left[P_t\right]}$$

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Calculating Certainty Equivalent Rate

$$\mathrm{E}\left[P_t\right]=\mathrm{E}\left[\exp\left(-\eta t\right)\right]\cdot \mathrm{E}\left[\exp\left(-\sum_{s=1}^{t}\varepsilon_s\right)\right]$$

$$\mathrm{E}\left[P_t\right]=\exp\left(-\bar{\eta}t+\frac{t^2\sigma_\eta^2}{2}\right)\times$$

$$\exp\left(\frac{\sigma_\xi^2}{2\left(1-\rho\right)^2}\left(t-\frac{2\left(\rho-\rho^{t+1}\right)}{1-\rho}+\frac{\rho^2-\rho^{2t+2}}{1-\rho^2}\right)\right)$$

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Results of analytic model

$$\tilde{r}_t = \bar{\eta} - t\sigma_\eta^2 - \sigma_\xi^2 \Omega(\rho, t)$$

$\tilde{r} =$ certainty-equivalent discount rate

$\bar{\eta} =$ mean discount rate

$\sigma_\eta^2 =$ variance of $\bar{r}$

$\sigma_\xi^2 =$ variance of ξ

$\rho =$ autocorrelation

$\Omega(\rho, t) =$ increasing function of ρ and t

Correlation Term

$$\Omega(\rho,t)=\frac{1-\rho^2+2\log(\rho)\rho^{t+1}\left(1+\rho-\rho^{t+1}\right)}{2\left(1-\rho\right)^3\left(1+\rho\right)}$$

Or, if $\rho = 1$:

$$\Omega(\rho,t)=\frac{1}{12}\left(1+6t+6t^2\right)$$

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Implications of model

- Certainty-equivalent rate declines from the mean rate as
  - forecast moves further into the future
  - uncertainty in the mean rate and deviations from the mean rate increase
  - persistence in deviations increases
- Significance of effect?
  -

$$\sigma_{\xi} = 0.23\%, \bar{\eta} = 4\%, \sigma_{\eta} = 0.52\%, \rho = 0.96$$

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