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National Emission Standards  
for Hazardous Pollutants**

**EPA 402-R-98-008**

**Background Information Document**

**Statistical Procedures for Certifying Phosphogypsum for Entry Into Commerce, As  
Required by Section 61.207 of 40 CFR Part 61, Subpart R**

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U.S. Environmental Protection Agency  
Office of Radiation and Indoor Air  
Washington, DC 20460**

## **PREFACE**

The Environmental Protection Agency (EPA) is promulgating revisions of 40 CFR Part 61, Subpart R, National Emission Standards for Radon Emissions from Phosphogypsum Stacks. This Background Information Document (BID) has been prepared in support of the final rulemaking. This BID contains an introduction, definitions of statistical terms, an overview, details of the methods for certifying phosphogypsum under procedures 1 and 2, and appendix explaining the statistics used for certifying phosphogypsum. Byron Bunger, an economist from EPA's Office of Radiation and Indoor Program developed and prepared this BID.

Copies of this BID, in whole or in part, are available to all interested persons. For additional information, contact Eleanor Thornton-Jones at (202) 564-9773 or write to:

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STATISTICAL PROCEDURES FOR CERTIFYING PHOSPHOGYPSUM  
FOR ENTRY INTO COMMERCE, AS REQUIRED BY SECTION 61.207  
OF 40 CFR PART 61, SUBPART R

Introduction

This document describes the methods that must be used for certifying phosphogypsum for entry into commerce under Sections 61.204 or 61.206, as required by Section 61.207 of 40 CFR part 61, subpart R—National Emission Standards for Radon Emissions From Phosphogypsum Stacks.

Section 61.204 allows for the lawful removal of phosphogypsum from a stack for agricultural purposes if the average radium-226 concentration at the location in the stack from which the material is to be removed does not exceed 10 pCi/g and if the administrative requirements of the section are satisfied. The demonstration that the average radium-226 concentrations does not exceed 10 pCi/g must be undertaken before the phosphogypsum can be removed from the stack and annually thereafter as long as the removal continues. Section 61.205 allows for the lawful removal of limited amounts of phosphogypsum from a stack for research and development purposes if the administrative requirements of the section are satisfied. This usage is not addressed in this document because the radium-226 concentration need not be reported under the provisions of this section. Section 61.206 governs the uses of phosphogypsum for any purpose other than those allowed by Sections 61.204 and Section 61.205. This section makes it unlawful to remove phosphogypsum from a stack for any purpose except those specified in Section 61.204 or Section 61.205 without the prior approval by EPA. Section 61.206 provides for the Administrator to allow for a specific use of phosphogypsum if the administrative requirements of the section are satisfied and if the applicant for the permit reports the average radium-226 concentration at the location in the stack from which the material is to be removed must be reported annually thereafter, as long as the removal continues.

This document provides the owner or operator of a phosphogypsum stack with the necessary techniques to demonstrate that the radium-226 concentration is below 10 pCi/g if the removal of phosphogypsum is pursuant to Section 61.204, and to report the radium-226 concentration of the stack if the removal of phosphogypsum is for the purposes pursuant to Section 61.206.

Definitions of Statistical Terms Used in this Discussion

*Critical value*—The per cent critical value of a probability distribution is that value above, or below, which only a per cent of the probability lies. Thus there is a .05 probability that a normally distributed variable will have a value above the upper 5% critical value, which is calculated by summing the product of 1.64 times the standards deviation of the distribution to the mean of the distribution. When testing an hypothesis, is the level of significance, and determines the critical value.

*Hypothesis testing*—A procedure for the statistical determination of the validity of an hypothesis. A test statistic, such as the standard normal variable, is calculated for the purpose of discriminating between a null hypothesis and an alternative.

*Level of significance*—The probability,  $\alpha$ , of rejecting the null hypothesis in test of an hypothesis.

*Sampling distribution*—The probability distribution assumed by a statistic such as the sample mean, calculated from a sample drawn from a population.

## Overview

The procedure for certifying phosphogypsum from an area of a stack for entry into commerce under the provisions of Sections 61.204 or 61.206 requires the collection of samples of phosphogypsum and the measurement of their radium-226 content. The samples must be collected from a regularly spaced locations across the area of the stack being considered for entry into commerce. After the radium-226 concentration in each sample is measured, the mean and standard deviation of the collected samples must be calculated.

If the phosphogypsum is to be removed under the provisions of Section 61.204, a test must be performed to demonstrate that the radium-226 concentration is below 10 pCi/g. A set of decision criteria, based on the sampling distribution for the sample mean, are used to determine if the phosphogypsum is acceptable for entry into commerce under this section. The use of these criteria require the determination of the critical value for 5% level of significance in the upper, or right hand, tail of the sampling distribution. The critical value is 95<sup>th</sup> percentile of the sampling distribution.

The decision criteria used for certification under Section 61.204 has three outcomes. If the critical value is less than or equal to 10 pCi/g, phosphogypsum from this area of the stack can be entered into commerce. If the mean of the collected samples is greater than or equal to 10 pCi/g, phosphogypsum from this area of the stack cannot be entered into commerce. If the sample mean is less than 10 pCi/g and the critical value is greater than 10 pCi/g, the phosphogypsum cannot be entered into commerce unless further testing demonstrates that a recalculated critical value is less than or equal to 10 pCi/g. The sample size must be increased, and the sample mean and standard deviation recalculated. The increase sample size reduces the standard deviation of the interval between the man of the sampling distribution and critical value. This increases the ability of the decision criteria to distinguish between the mean of the sample and the 10 pCi/g concentration limit, which improves the chance that the radium-226 concentration can be shown to be less then 10 pCi/g. A calculation is performed to determine the sample size needed to reduce the recalculated critical value to a value no greater than 10 pCi/g.

If a larger sample size is needed to demonstrate that the sample mean is less than 10 pCi/g, the number of additional samples required increases rapidly as the mean approaches 10 pCi/g, and can be quite large in cases where the sample mean is only slightly less than 10 pCi/g. In such cases the cost of gathering additional samples and measuring their radium-226

concentrations may become a factor in the decision to continue with the attempt to enter the phosphogypsum from this area of the stack into commerce for agricultural purposes.

Any additional samples must also be taken from regularly spaced locations across the area of the phosphogypsum stack being considered for entry into commerce. Once the required number of additional samples have been collected, the radium-226 concentrations in each sample must be measured. The mean and standard deviation of the radium-226 concentrations for the entire set of sample concentrations (including those previously measured) must be recalculated and a new sampling distribution established. The critical value for a 5% level of significance in the upper tail of the new sampling distribution is established. The decision criteria must then be revisited. As before, phosphogypsum from this area of the stack can be entered into commerce only if the critical value is less than or equal to 10 pCi/g.

Although acceptance for entry into commerce is the objective of increasing the sample size and establishing the new sampling distribution and critical value, and is the expected outcome of the reconsideration, it is possible the recalculated critical value will not be less than or equal to 10 pCi/g. This is because random variation in the new sample concentrations, which can result from nonuniformity in the distribution of radium-226 may cause an increased sample mean or standard deviation. Either or both of these increases can cause the recalculated critical value to be larger than had been projected, causing it to be greater than 10 pCi/g. If this is the case either the sample size must be increased once again, and a new sampling distribution and critical value determined, or the attempt to certify that area of the stack for entry into commerce must be abandoned.

The details of the method for certifying phosphogypsum for entry into commerce under Section 61.204 is described in Procedure 1 below. The statistical methods that underlie Procedure 1 are discussed in the Appendix.

If phosphogypsum is to be removed from the stack under the provisions of Section 61.206, the mean, standard deviation, sample size, and 95<sup>th</sup> percentile must be reported. The method to be used for their calculation is described in Procedure 2 below.

#### Procedure 1: Certification of Radium-226 Concentrations Under Section 61.204

Section 61.204 provides for the distribution and use of phosphogypsum for agricultural purposes. Procedure 1 describes the sampling techniques required to satisfy the provisions of Section 61.207, when phosphogypsum is distributed and used under Section 61.204.

The owner or operator of the phosphogypsum stack must demonstrate that the 95<sup>th</sup> percentile of the sample mean for radium-226 is no greater than 10 pCi/g if the phosphogypsum is to be introduced in commerce under the provisions of Section 61.204. Steps 1 through 5 are required of all applicants. Depending on the outcome of the decision criteria in step 5, further analysis may be required in order to meet the conditions for certification.

- (1) A minimum of 30 phosphogypsum samples shall be taken at regularly spaced intervals across the surface of the location on the stack from which the phosphogypsum will be removed. Let  $n_1$  represent the number of samples taken.
- (2) Measure the radium-226 concentration of each of the  $n_1$  samples in accordance with analytical procedures described in 40 CFR part 61, appendix B, Method 114.
- (3) The mean,  $\bar{x}_1$ , and the standard deviation,  $s_1$ , of the  $n_1$  radium-226 concentrations must be calculated:

$$\bar{x}_1 = \frac{\sum_{i=1}^{n_1} x_i}{n_1},$$

$$s_1 = \sqrt{\frac{\sum_{i=1}^{n_1} (x_i - \bar{x}_1)^2}{n_1 - 1}},$$

where  $\bar{x}_1$  and  $s_1$  are expressed in pCi/g.

- (4) Calculate the 95<sup>th</sup> percentile for the distribution,  $\bar{x}^*$ , using the following equation:

$$\bar{x}^* = \bar{x}_1 + 1.64 \left( \frac{s_1}{\sqrt{n_1}} \right),$$

where  $\bar{x}^*$ , is expressed in pCi/g.

- (5) Based on the values for  $\bar{x}_1$  and  $\bar{x}^*$ , one of the following conditions will be met:
  - (a) If  $\bar{x}_1 < 10$  pCi/g and  $\bar{x}^* \leq 10$  pCi/g; phosphogypsum can be removed from this area of the stack for distribution to commerce pursuant to Section 61.204. Report the mean, standard deviation, sample size, and 95<sup>th</sup> percentile.
  - (a) If  $\bar{x}_1 > 10$  pCi/g; phosphogypsum cannot be removed from this area of the stack for distribution to commerce pursuant to Section 61.204. This ends the procedure. Certification for this area of the stack is not possible.
  - (b) If  $\bar{x}_1 < 10$  pCi/g and  $\bar{x}^* > 10$  pCi/g, the decision criteria are inconclusive. Further sampling is required to determine phosphogypsum can be removed from this area

of the stack. The objective of the following procedure is to demonstrate, with 95% probability, that the phosphogypsum from this area of the stack has a radium-226 concentration no greater than 10 pCi/g. The procedure is iterative, the sample size may have to be increased more than on time. The following procedures must be followed if the attempt to certify the phosphogypsum for entry into commerce is to be continued; otherwise the phosphogypsum cannot be removed from this area of the stack for distribution to commerce under the provisions of Section 61-204. Proceed with steps (6) through (11) below.

- (6) Solve the following equation for the total number of samples required:

$$n_2 = \left( \frac{1.64s_1}{10 - \bar{x}_1} \right)^2 \cdot$$

The sample size  $n_2$  shall be rounded upwards to the next whole number. The number of additional samples needed is  $n_A = n_2 - n_1$ .

- (7) Obtain the necessary number of additional samples,  $n_A$ , which shall also be taken at regularly spaced intervals across the surface of the location on the stack from which phosphogypsum will be removed.
- (8) Measure the radium-226 concentration of each of the  $n_A$  additional samples in accordance with the analytical procedures described in 40 CFR part 61, appendix B, Method 114.
- (9) Recalculate the mean and standard deviation of the entire set of  $n_2$  radium-226 concentrations by joining this set of  $n_A$  concentrations with the  $n_1$  concentrations previously measured. Use the formulas in step (3), substituting the entire set of  $n_2$  samples in place of the  $n_1$  samples called for in step (3), thereby determining the mean,  $\bar{x}_2$ , and standard deviation,  $s_2$ , for the entire set of  $n_2$  concentrations.
- (10) Repeat the procedure described in step (4), substituting the recalculated mean,  $x_2$  for  $x_1$ , the recalculated standard deviation,  $s_2$ , for  $s_1$ , and total sample size,  $n_2$ , for  $n_1$ .
- (11) Repeat the procedure described in step (5), substituting the recalculated mean  $\bar{x}_2$  for  $\bar{x}_1$ . The test in step (5) may, again, be inconclusive, resulting in outcome (c). This can result from nonuniformity in the distribution for radium-226 in phosphogypsum and from the random nature of radiation decay. If this is the case, the sample size must again be increased, and steps (6) through (11) repeated; otherwise the attempt to certify phosphogypsum from this area of the stack must be abandoned.

#### Procedure 2: Sampling Radium-226 Concentrations Under Section 61.206

Uses of phosphogypsum other than those allowed by Sections 61.204 and 61.205 are governed by the requirements of Section 61.206. Procedure 2 describes the sampling techniques

required to satisfy the provisions of Section 61.207, when phosphogypsum is distributed and used under Section 61.206.

A minimum of 30 phosphogypsum samples shall be taken at regularly spaced intervals across the surface of the location on the stack from which the phosphogypsum will be removed.

Measure the radium-226 concentration of each of the samples in accordance with the analytical procedures described in 40 CFR part 61, appendix B, Method 114.

The mean,  $\bar{x}$ , and the standard deviation,  $s$ , of the radium-226 concentrations must be calculated:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n},$$

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}},$$

where  $n$  is the sample size, and  $\bar{x}$  and  $s$  are expressed in pCi/g.

Calculate the 95<sup>th</sup> percentile for the distribution,  $\bar{x}^*$ , using the following equation:

$$\bar{x}^* = \bar{x} + 1.64 \left( \frac{s}{\sqrt{n}} \right),$$

where  $\bar{x}^*$  is expressed in pCi/g.

Report the mean, standard deviation, sample size, and 95<sup>th</sup> percentile.

## APPENDIX

Explanation of the Statistics Used for Certifying  
Phosphogypsum for Agricultural Purposes Under  
the Provisions of Section 61.204,  
40 CFR Part 61, Subpart R

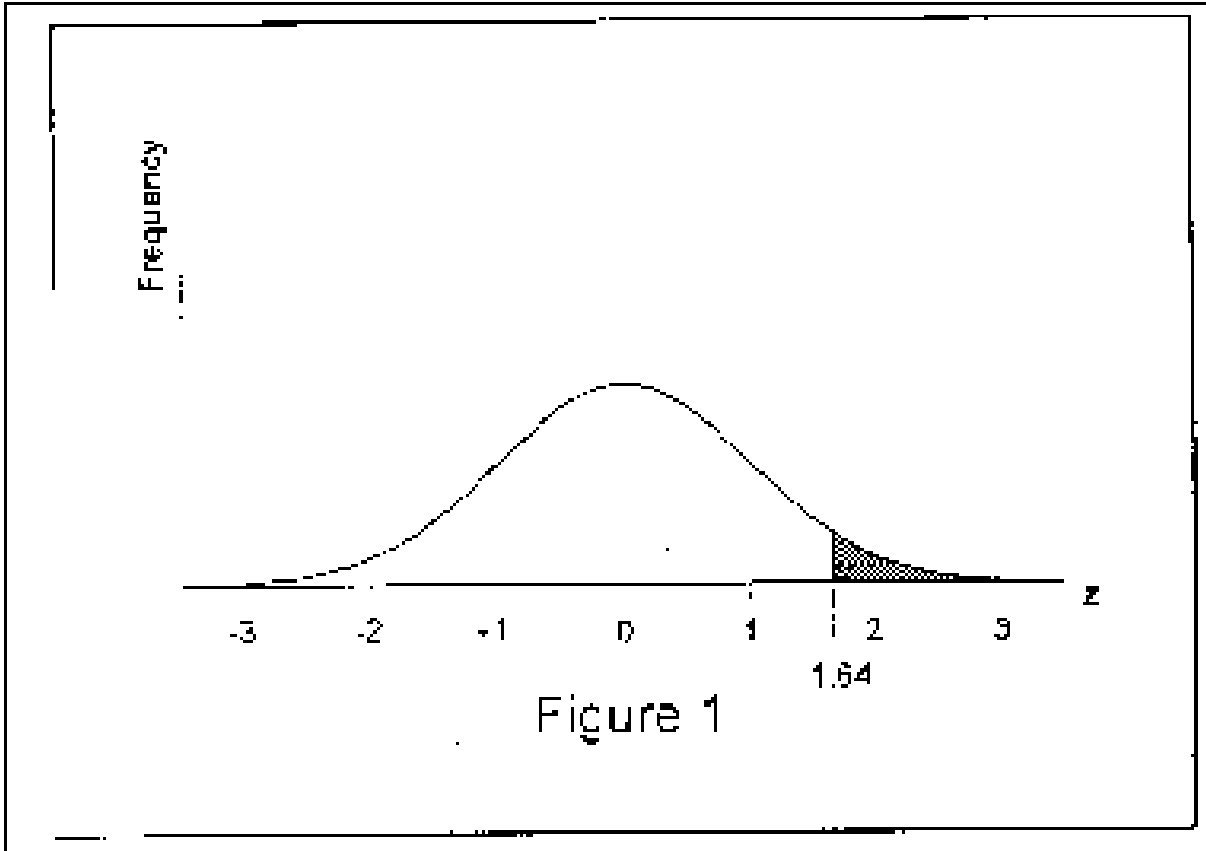


A series of fundamental theorems in statistical theory proves that the distribution of the mean of a sample drawn from any population will approach the normal distribution as the sample size increases without limit. The sample mean approaches that of the population from which the sample is drawn, and its variance approaches that of the population divided by the sample size. The result is that the expected value of  $\bar{x}$  is  $\mu$ , the population mean, and  $\bar{x}$  can be assumed to be normally distributed if estimated with a sufficiently large sample. (The expected value is a statistical term for the average value of the variable over all possible values of the variable.) The variance of the distribution of  $\bar{x}$  is  $\sigma^2/n$ , where  $\sigma^2$ , is the population variance. Also, it can be shown that the sample variance,  $s^2$ , more closely approximates the population variance,  $\sigma^2$ , as the sample size increases.

Although all of these results are stated in terms of improved approximations as the sample size increases, a minimum sample size of 30 is usually considered large enough that the approximations can be assumed to hold.

The outcome is that the mean of a sample of 30 or more radium-226 concentrations can be assumed to be normally distributed with a mean approximately equal the true radium concentration,  $\mu$ , and a variance approximately equal the true variance,  $\sigma^2$ , divided by the sample size,  $n$ .

Figure 1 shows the normal distribution (or density function) with a mean of zero and a standard deviation of one. (The standard deviation is the square root of the variance.) This particular density function is known as the standard normal distribution. The areas and ordinates of this distribution are found in standard statistical tables. The letter  $z$  is a symbol often used for the variate of this particular distribution. The value of the mean or any percentile (such as the 95<sup>th</sup> percentile, which determines the upper 5% tail) from any normal distribution can be transformed into a corresponding value for the standard normal distribution and then transferred back to the original distribution. This makes it possible to use the tables for the standard normal distribution to determine the percentiles for any normally distributed variate. The normal distribution has infinitely long tails on the left and right; and the area under this distribution, when integrated from negative infinity to positive infinity is one. Therefore, the probability of the occurrence of  $z$  within any specified range is determined by integrating the area under the normal distribution within this range. The areas under the tails of this distribution are most often of interest in testing hypotheses. The right tail, shown by the shaded area in Figure 1, includes all values of  $z$  from 1.64 to infinity. It has an area of 0.05. The value of  $z$  equal 1.64 is the 95<sup>th</sup> percentile. In sampling from this distribution, the probability of any single value of  $z$  equaling or exceeding 1.64 is 5%.



A major difficulty in determining whether or not the true radium-226 concentration in phosphogypsum from an area of a stack is less than or equal 10 pCi/g is the random variation in the measured values. This randomness is caused by the uneven distribution of radium-226 in the phosphogypsum and the randomness inherent in the measurement of the radium concentration in individual samples. The true concentration can never be exactly determined because of this randomness. The accuracy of the estimated concentrations can be improved upon only by increasing the number of samples used in the estimation.

The decision making framework for determining whether or not phosphogypsum from an area of a stack can be entered into commerce for agricultural purposes must be designed in recognition of this randomness. The method used for structuring this decision making is based on a test of hypothesis. The test is designed to accept a hypothetical outcome, or null hypothesis, unless there is persuasive evidence that this outcome is incorrect. Depending on the purpose of the test, the null hypothesis may be defined to be the expected or usual outcome; or, as in this case, the outcome that is not desired. Therefore, the null hypothesis represents the case where phosphogypsum is unacceptable for agricultural purposes, and represents a radium-226 concentration greater than 10 pCi/g.

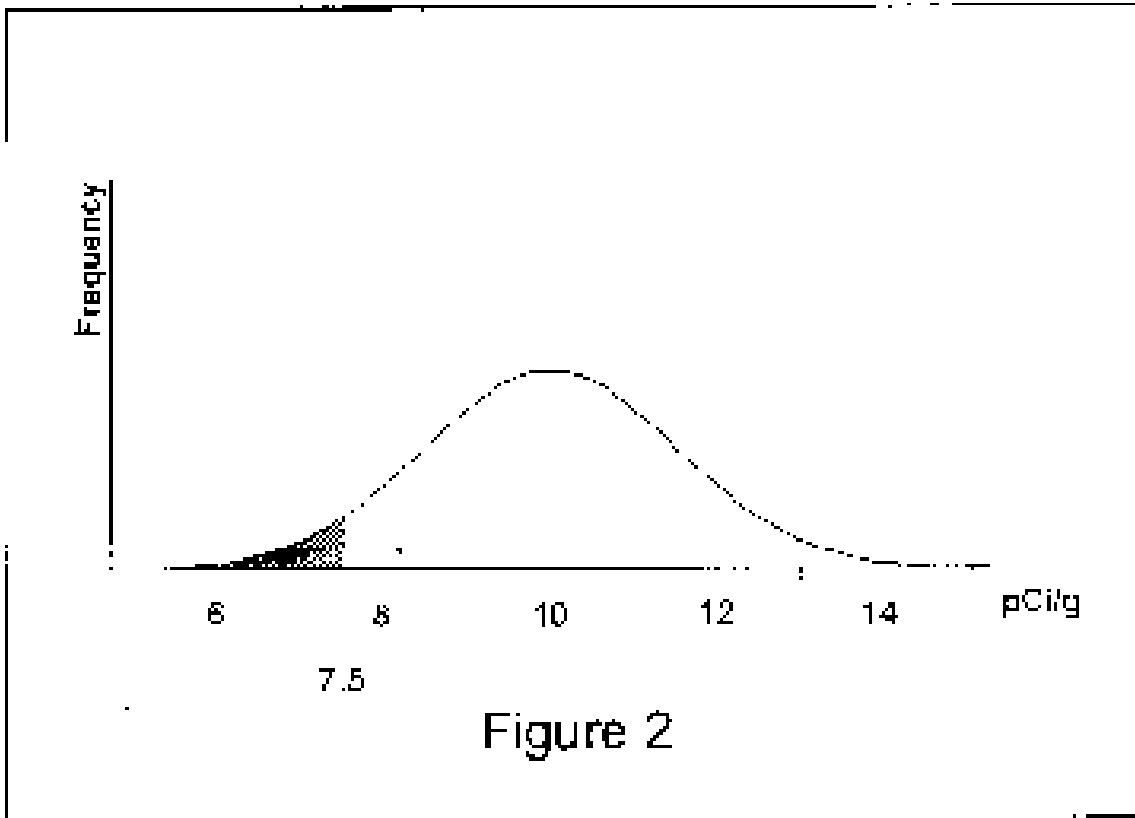
The null hypothesis is rejected only if the evidence is persuasive that the true concentration is less than 10 pCi/g. This means that the phosphogypsum is determined to be acceptable for agricultural purposes only if the radium-226 concentration can be shown to be less than 10 pCi/g by means of the statistical test described in this document.

To illustrate hypothesis testing, assume an area of a hypothetical phosphogypsum stack has a true concentration equal 10 pCi/g. This concentration is used because it is the highest concentration permissible for use in agriculture, and therefore, the highest concentration not meeting the conditions of the null hypothesis. The distribution of the mean of a sample drawn from this area would be expected to be normally distributed with a mean of 10 pCi/g. The standard deviation of this distribution could be estimated from a sample drawn from this area (when dealing with the distribution of the sample mean, the standard deviation is usually termed the standard error). Assume, for this illustration, that the standard error is 1.5 pCi/g. (A sample of 30 with a standard deviation equal 8.2 pCi/g would yield a standard error of the mean equal 1.5 pCi/g.) The standard deviations of radium-226 measurements on five phosphogypsum stacks, as reported in the document: [A Long-Term Study of Radon and Airborne Particulates at Phosphogypsum Stacks in Central Florida](#), (EPA 520/5-88-021, Oct. 1988), range from about 1.65 to 18.3 pCi/g, so a standard deviation equal 8.2 pCi/g is reasonable. The sampling distribution is shown in Figure 2. This distribution is entirely hypothetical, and is used only to illustrate this discussion. The shading in the left hand tail is explained below.

When testing an area of the stack with an unknown radium concentration, the mean of the sample can be compared to this distribution to determine how likely it is that the sample was drawn from an area of a phosphogypsum stack with a mean concentration greater than 10 pCi/g. The null hypothesis would be rejected only if the sample mean were small enough that it is unlikely that it could have been drawn from an area with a true concentration greater than 10 pCi/g.

The probability of erroneously rejecting the null hypothesis expresses the likelihood of mistakenly concluding that phosphogypsum can be entered into commerce when, in fact its radium-226 concentration is greater than 10 pCi/g. This probability must be established before the null hypothesis can be tested, and, when decided upon, determines the critical value. The critical value divides the sampling distribution into a region for accepting the null hypothesis and a region for rejecting the null.

There are no rules for determining an acceptable probability of erroneously rejecting the null hypothesis. The choice should be made in the context of the consequences of an incorrect decision. If there are expected to be substantial consequences from incorrectly rejecting the null (i.e., concluding the concentration is less than or equal 10 pCi/g when it is, in fact, greater than 10 pCi/g), then it is reasonable to establish a small chance of error. If the consequences are not expected to be substantial, it is reasonable to establish a larger chance of error.



Continuing with the illustration, assume a 5% probability is decided upon. This establishes a critical value equal 7.5 pCi/g, which is calculated as follows:

$$\begin{aligned}\bar{x}_c &= \mu - z \left( \frac{s}{\sqrt{n}} \right), \\ &= 10 - 1.645 (1.5), \\ &= 7.5,\end{aligned}$$

where  $\bar{x}_c$  is the critical value,

$$\mu = 10 \text{ pCi/g},$$

$z = 1.64$ , based on a 5% probability of erroneously rejecting the null hypothesis,

and  $s/\sqrt{n} = 1.5 \text{ pCi/g}$ .

The null hypothesis is accepted if the mean of a sample drawn from an area of a pile

being considered for entry into commerce for agricultural purposes is greater than 7.5 pCi/g, and is rejected if the sample mean is less than or equal 7.5 pCi/g. The region of rejection is the shaded area to the left of 7.5 pCi/g in Figure 2.

Therefore, if a sample drawn from an area of a stack with an unknown concentration has a mean less than or equal 7.5 and a standard deviation of 8.2, the null hypothesis would be rejected, which means that the phosphogypsum is judged to be acceptable for use for agricultural purposes.

Although this illustration is a straightforward application of a test of an hypothesis, the test can be structured somewhat more conveniently with some recalculation. This is done by determining a different critical value,  $\bar{x}^*$ , based on the sampling distribution for sample drawn from the area of the stack being tested. The new critical value is calculated using the following equation:

$$\bar{x}^* = \bar{x} + z \left( \frac{s}{\sqrt{n}} \right),$$

where  $\bar{x}^*$  is the 95<sup>th</sup> percentile, which determines the upper 5% tail area,

$\bar{x}$  is the mean of the measured radium concentration (in pCi/g) measured from the  $n$  samples,

$s$  is the standard deviation of the measured radium concentration (in pCi/g) measured from the  $n$  samples,

and  $z = 1.64$ , based on an upper tail area equal 5%.

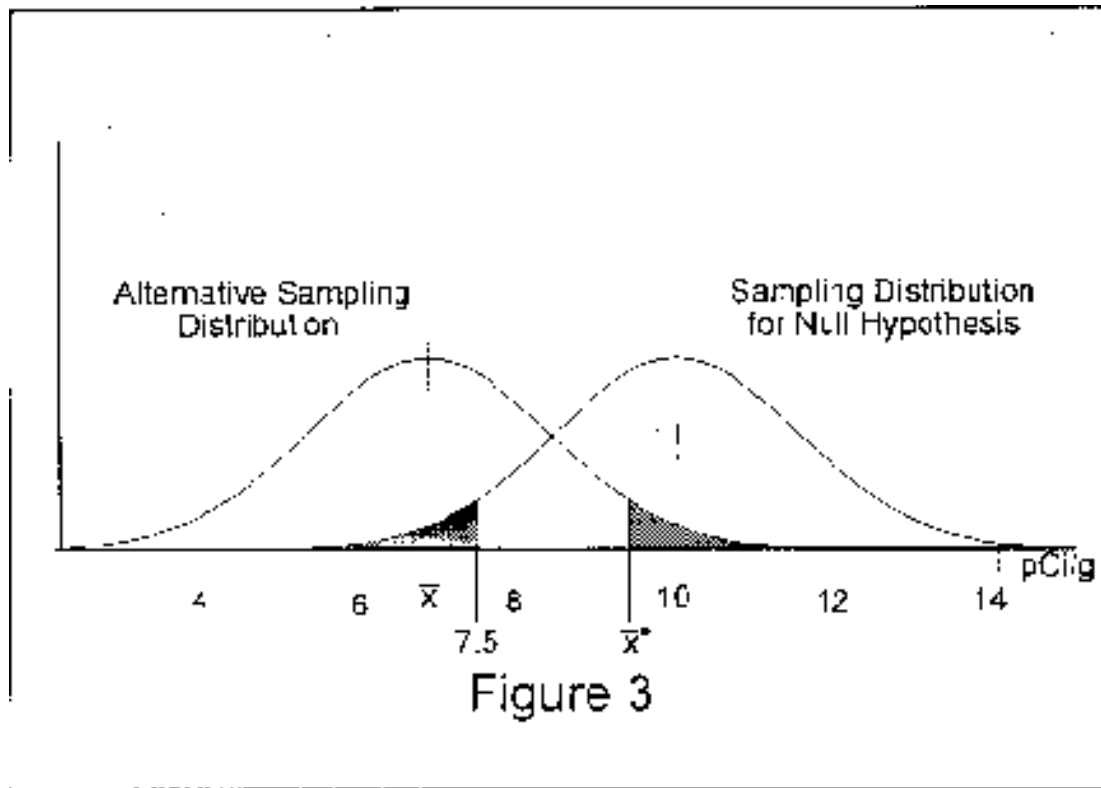
The critical value has been redefined, and has changed numerical value. Nevertheless, the new decision rule has the same outcome as the previously formulated decision rule. This is shown in the following equations:

if 
$$\bar{x} \leq 10 - z \left( \frac{s}{\sqrt{n}} \right),$$

then 
$$\bar{x} + z \left( \frac{s}{\sqrt{n}} \right) \leq 10,$$

or 
$$\bar{x}^* \leq 10.$$

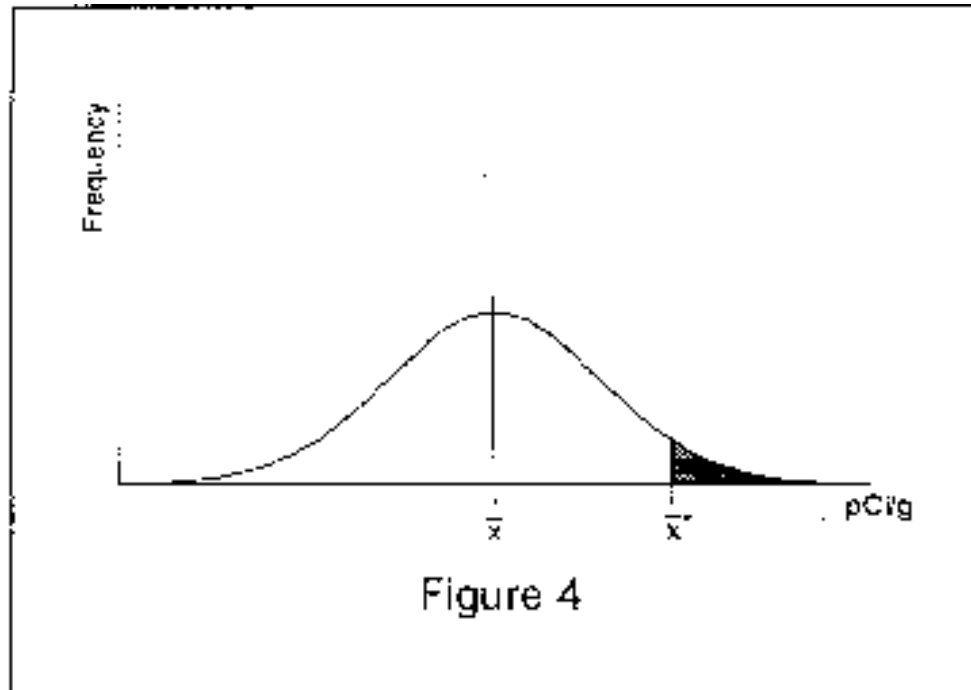
The equivalency of these two approaches can be seen in Figure 3 where the null hypothesis is represented by the distribution on the right. The region below 7.5 pCi/g, under the left tail of



this distribution, represents the rejection region for the null hypothesis using the first definition of the critical value. The distribution on the left illustrates an alternative sampling distribution with a mean less than 7.5 pCi/g. The mean of the alternative distribution falls in the region for rejecting the null hypothesis. A radium-226 concentration of 10 pCi/g falls in the upper tail of the alternative distribution. The alternative critical value,  $\bar{x}^*$ , bounds the area in the right hand tail of the alternative distribution that represents rejection of the alternative hypothesis (and the acceptance of the null hypothesis). The shaded area to the right of  $\bar{x}^*$  is this region, and  $\bar{x}^*$  divides the alternative sampling distribution into acceptance and rejection regions. Based on the second method for determining the critical value, the null hypothesis is rejected since 10 pCi/g is greater than  $\bar{x}^*$ .

The sampling distributions for the null hypothesis and for the alternative have the same standard deviation. Considering how the critical value,  $\bar{x}^*$  would move to the right if the mean of this alternative sampling distribution were moved to the right confirms that a sampling distribution with a mean of 7.5 pCi/g would have a critical value,  $\bar{x}^*$  falling at 10pCi/g, as proven on the previous page.

Figure 4 shows only the alternative sampling distribution. The decision rule constructed thus far states that a sample found to have an  $\bar{x}^*$  less than or equal to 10 pCi/g would be judged to have a true radium-226 concentration less than 10 pCi/g and a sample with an  $\bar{x}$  greater than 10 pCi/g would be judged to have a true concentration greater than 10 pCi/g.



However, this decision rule is not complete because it does not address a third possibility; that  $\bar{x}$  is less than 10 pCi/g while  $\bar{x}^*$  is greater than 10 pCi/g. Even though  $\bar{x}$  is less than 10 pCi/g in this case, it is not sufficiently low that there is assurance that the true concentration is below 10 pCi/g. Further sampling to increase the sample size is a way of reducing the interval between  $\bar{x}$  and  $\bar{x}^*$ . This is because the variance of the distribution of the sample mean is inversely related to the sample size; therefore, increasing the sample size reduces the variance of the sampling distribution, as discussed on the first page of this appendix. With a sufficiently large sample this interval can be reduced enough that  $\bar{x}^*$  is equal to or less than 10 pCi/g. Therefore, increasing the sample size increases the ability of the decision rule to distinguish between distributions having different means, increasing the likelihood of either accepting or rejecting the null hypothesis.

Figure 5 demonstrates this; the wider, flatter, distribution could represent the sampling distribution for the mean concentration based on a sample size 30, and the distribution with the higher, narrower, peak could represent the distribution of  $\bar{x}$  for a four fold increase in the sample size to 120, which reduces the standard deviation by a factor of 2. In the case shown

there, the increased sample size reduces the standard deviation of the sampling distribution sufficiently that the new critical value,  $\bar{x}^*$ , is less than 10 pCi/g.

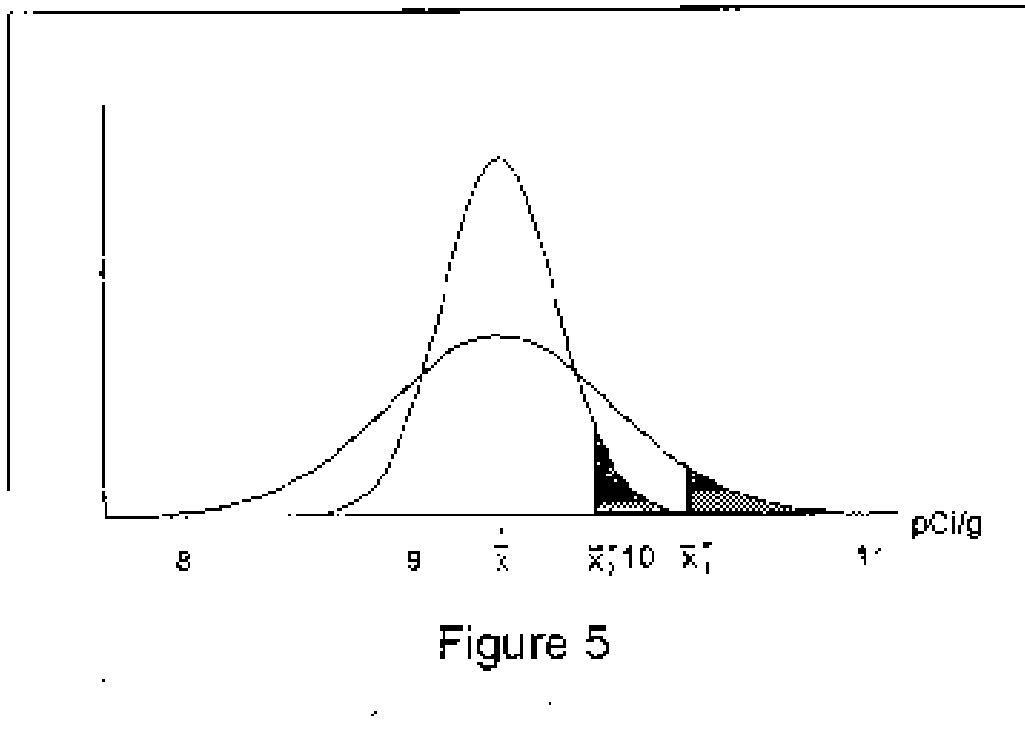


Figure 5

This is the basis for the provision in Section 61.207(c), which requires further testing by increasing the sample size, as discussed in Procedure 1, steps (6) through (11), on p. 6.

In one sense, Figure 5 is an over simplification because it shows the two distributions with the same mean. This is unlikely to occur in practice because any change in the individual members of a sample (here an increase in sample size), would be expected to cause changes in both the sample mean, and the standard deviation, due to random variations in the observations included in the sample. Therefore, the mean of the new sampling distribution would not be expected to be exactly the same as the mean of the original sampling distribution, nor would the standard deviation of the new sampling distribution be expected to be exactly one half the standard deviation of the original to be exactly one half the standard deviation of the original sampling distribution.

The result is that the increased sample size required in Section 61.207(c), and described in Procedure 1, steps (6) through (11), does not guarantee that the recalculated value for  $\bar{x}^*$  will be below 10 pCi/g. The recalculated sample mean may be larger than the mean previously calculated, or the recalculated sample standard deviation may not be as small as expected. If the recalculated  $\bar{x}^*$  is not less than or equal 10 pCi/g the sample size can again be increased and the



decision rule reconsidered. In practice the cost of continuing to take samples and revisiting the decision rule may become prohibitive and the attempt to certify this area of the stack for the removal of phosphogypsum for agricultural purposes may be abandoned.

Clearly, the closer the true radium concentration is to 10 pCi/g, the greater the required reduction in the interval between  $\bar{x}$  and  $\bar{x}^*$ ; therefore, the greater the required reduction in the variance of the sample mean and the larger the size of the sample required. Again, the cost of greatly increasing the sample size may become prohibitive, and the attempt to certify this area of the stack for distribution to commerce under the provisions of Section 61.204 may be abandoned

An area of a phosphogypsum stack is rejected for distribution to commerce under the provisions of Section 61.207(b)(3), described in Procedure 1, step (5)(b)) on p. 5, if the mean of the sample is equal 10 pCi/g, with no provision for reconsideration using an increased sample size. The reason is that the only way  $\bar{x}^*$  could be made equal 10 pCi/g if  $\bar{x}$  is found to be equal 10 pCi/g is that the sample size must be increased without limit. Since an infinitely large sample is not possible, this area of the stack cannot be show to have an  $\bar{x}^*$  less then or equal 10 pCi/g.