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Alternative Presentation of PM Expert Elicitation Results

Presentation to EPA Science Advisory Board 812 Council Health Effects Subcommittee

December 15,2009

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Background

- Major Expert Elicitation (EE) study of the mortality effects of PM2.5 completed by EPA in 2006.
- Elicited subjective probabilistic distributions of uncertainty in PM-mortality concentration response coefficient for use in EPA benefits analyses.
- 12 experts, 12 distributions (A - L)
- Individual expert distributions programmed in BenMAP, applied (unpooled) in subsequent analyses (PM, NAAQS RIA; RSM-based PM co-benefits in other RIAs).
- Lack of combined estimate poses presentation challenges
  - Reporting of 12 distributions can be cumbersome.
  - SAB critiqued EE range reported in PM NAAQS as misleading.

Past SAB Advice

- Excerpt from EPA SAB PM NAAQS RIA consultation in 2008:
  - "Where experts largely agree, it would be appropriate to collapse the various estimates into a single distribution (or point estimate with uncertainty bounds) while still providing the individual estimates elsewhere…In future analyses, the decision about aggregation must be made in the context of each analysis and its purpose."
- Is aggregation a reasonable approach for the 812 analysis? Is there a viable means of combining the PM EE results?

Challenges

- PM EE study not designed to yield "combinable" estimates
  - No test or "seed" questions in protocol
  - No self- or peer-weights
  - Consensus not an objective
  - Allowed for variation in:
    - Shape of C-R function
    - Threshold
    - Treatment of Causal Probability
  - Likely significant dependence among expert responses.

Options for Combining Results

- Substantial literature from 80s onward (Genest and Zidek, Clemen and Winkler, Cooke, Jouini and Clemen) but little agreement on whether and how to combine distributions mathematically
- Choices
  - Linear opinion pool
  - Logarithmic opinion pool
  - Cooke's classical method
  - Copula functions

Opinion Pooling

- Linear opinion pool

$$f(\theta) = \sum_{i=1}^{n} w_i f_i(\theta)$$

  - Weighted average of individual distributions using subjective weights (e.g., equal weighting
  - Useful where other weights are lacking
  - Equal weights potentially appropriate for public policy analysis
  - Can perform as well as more complex methods (Clemen, 1989)
  - Does not account for dependence among experts (may overweight some views)
  - Tends to broaden distributions

Opinion Pooling (cont'd)

- Logarithmic Opinion Pool

$$f(\theta) = k\prod_{i=1}^{n} f_i(\theta)^{w_i}$$

  - Derives a combined distribution by taking a weighted geometric mean of a set of individual distributions
  - Weights can be subjective, including equal weights
  - Not designed to address dependence among experts
  - "Single Expert Veto": any values considered implausible by any one expert are zeroed out in the pooled distribution (O'Hagan et al., 2006)
  - Tends to produce narrower distributions, projecting greater knowledge
  - Rarely used

Other Approaches

- Cooke's method
  - Requires performance measures based on responses to seed questions
- Copula functions
  - First proposed by Jouini and Clemen (1996); Also Hammitt and Shlyakhter, 1999).
  - A copula is "a mathematical function that can be used to represent probabilistic dependence when coupling marginal probability distributions (the experts' judgments) into a multivariate distribution (the joint likelihood of the experts' judgments)." (Hammitt and Shlyakhter, 1999).
  - Flexible; does not restrict the form of the expert distributions
  - Incorporates dependence among experts
  - Can exhibit the single-expert veto

Example Application of Copula Function

- Many copula functions exist. We used same form as Hammitt and Shlyakhter and Jouini and Clemen:

$$f_n(\theta) = kC_{n|\alpha}\,[1 - H_1(\theta), 1 - H_2(\theta), \ldots, 1 - H_n(\theta)]h_1(\theta)h_2(\theta)\ldots h_n(\theta) \quad (1)$$

$$C_{n|\alpha}(u_1, u_2, \ldots, u_n) = \log_\alpha\left[1 + \frac{(\alpha^{u_1} - 1)\ldots(\alpha^{u_n} - 1)}{(\alpha - 1)^{n-1}}\right] \quad (2)$$

- Where:
  - $H_i(\theta)$ = expert i's CDF, evaluated at θ
  - $h_i(\theta)$ = expert i's PDF, evaluated at θ
  - α = measure of dependence (0 =complete dependence; 1 = complete independence)
  - n = number of experts
  - k = normalization constant
  - All experts treated as equally dependent or independent

Approach

1. **Derive PDFs/CDFs for C-R coefficients.** Obtain mathematical expression of $h_i(\theta)$ and $H_i(\theta)$ for each expert.
2. **Input PDFs/CDFs into copula.** Evaluate across range of thetas.
3. **Normalize copula.** Set k so area under curve = 1.
4. **Make BenMAP compatible.** Convert function for input into BenMAP.
5. **Repeat for different baseline PM levels**
   - **PM >16 $\mu g/m^3$**
   - **10 <PM $\leq$ 16 $\mu g/m^3$**
   - **7 <PM $\leq$ 10 $\mu g/m^3$**
   - **PM $\leq$ 7 $\mu g/m^3$**
6. **Run BenMAP.** Pool Copula results across baseline PM levels.

Derivation of PDFs and CDFs

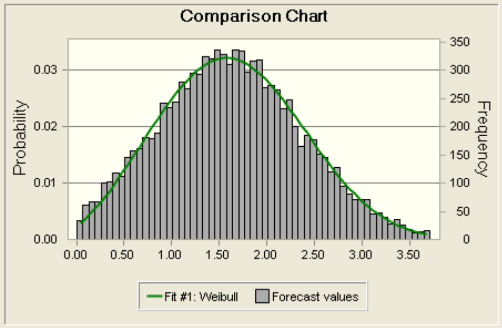
- Challenges
  - Some experts provided fractiles (as requested) of an unspecified distributional form.
  - Even experts who specified parametric distributions modified them in some way.
    - Some are truncated.
    - About half the experts gave distributions conditional on a causal relationship.
    - One expert specified a probabilistic threshold.
- The Good News
  - Re-ran 812 CMAQ core scenario results through BenMAP with no threshold configuration for expert K. *Results differ only minimally from applying threshold.* Can reasonably assume no threshold for this application.

Derivation of PDFs and CDFs (cont'd)

- Used Crystal Ball™ to:
  1. sample from elicited distributions (n = 10,000)
  2. Fit distributions to sample output

A

10,000 Values | Split View | 9,974 Displayed



Ranked by: Chi-Square

Distribution	A-D	Chi-Square	K-S	Parameters
Weibull	1.6001	73.6200	.0090	Location=-0.34,Scale=2.22,Shape=2.89197
Beta	1.7228	99.6532	.0097	Minimum=-0.64,Maximum=5.04,Alpha=5.16504,
Normal	7.4210	270.0900	.0148	Mean=1.63,Std. Dev.=0.75
Gamma	19.4342	279.5472	.0291	Location=-1.00,Scale=0.22,Shape=11.72222
Student's t	10.4659	376.7536	.0240	Midpoint=1.63,Scale=0.73,Deg. Freedom=10.99
Logistic	15.6636	489.8996	.0236	Mean=1.62,Scale=0.43
Triangular	117.5299	674.3816	.0783	Minimum=-0.02,Likeliest=1.23,Maximum=4.03
Max Extreme	63.6329	780.7788	.0504	Likeliest=1.26,Scale=0.68
Min Extreme	148.3011	1,599.5740	.0721	Likeliest=2.01,Scale=0.76
BetaPERT	343.9256	2,133.8540	.1037	Minimum=-0.64,Likeliest=1.35,Maximum=5.04
Lognormal	292.0435	2,637.6312	.1134	Mean=1.73,Std. Dev.=1.24
Uniform	1,064.369	4,921.5672	.2472	Minimum=0.00,Maximum=4.00
Exponential	1,240.274	7,069.6244	.2514	Rate=0.61
Pareto	3,758.557	88,199.1840	.5013	Location=0.00,Shape=0.16644

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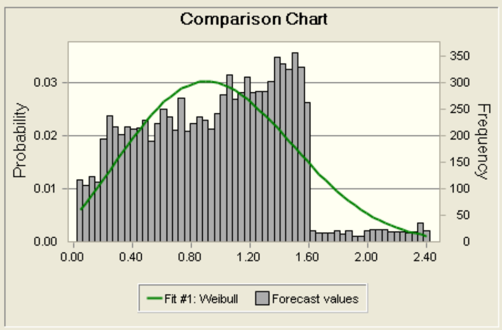
F

(high)

10,000 Values | Split View | 9,874 Displayed



Ranked by: Chi-Square

Distribution	A-D	Chi-Square	K-S	Parameters
Weibull	60.1377	1,957.4380	.0769	Location=-0.22,Scale=1.37,Shape=2.53059
Normal	59.4432	2,225.9248	.0639	Mean=0.98,Std. Dev.=0.52
Beta	66.8627	2,260.4532	.0686	Minimum=-1.20,Maximum=9.36,Alpha=13.9721
Gamma	90.1294	2,372.6224	.0728	Location=-0.65,Scale=0.17,Shape=9.60227
Logistic	73.2959	2,667.7344	.0594	Mean=0.98,Scale=0.30
Max Extreme	125.7130	2,868.5408	.0881	Likeliest=0.73,Scale=0.46
Triangular	180.2835	3,128.2216	.1406	Minimum=0.01,Likeliest=0.31,Maximum=2.72
Student's t	172.7694	3,138.1524	.0801	Midpoint=0.98,Scale=0.43,Deg. Freedom=30
Min Extreme	224.1530	3,383.8176	.1067	Likeliest=1.25,Scale=0.56
Lognormal	352.9511	5,852.0432	.1275	Mean=1.06,Std. Dev.=0.92
Uniform	1,496.921	6,260.0644	.3587	Minimum=0.02,Maximum=2.70
Exponential	909.4712	7,809.5800	.1935	Rate=1.02
BetaPERT	1,993.243	18,775.2700	.3431	Minimum=-1.20,Likeliest=-0.57,Maximum=9.36
Pareto	3,020.742	50,213.5780	.4170	Location=0.02,Shape=0.27167

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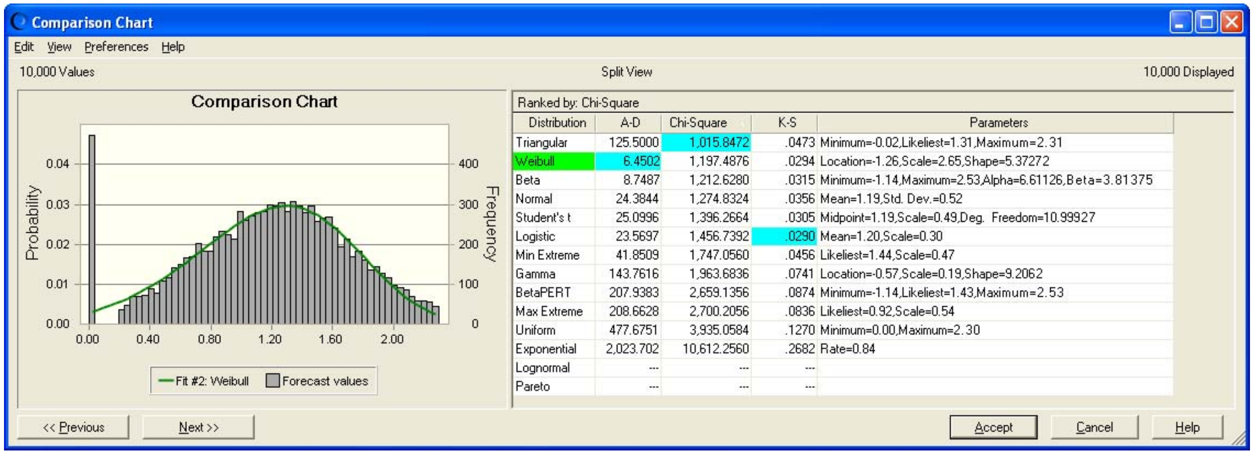
- For conditional distributions, we opted not to incorporate p(causal) before fitting.

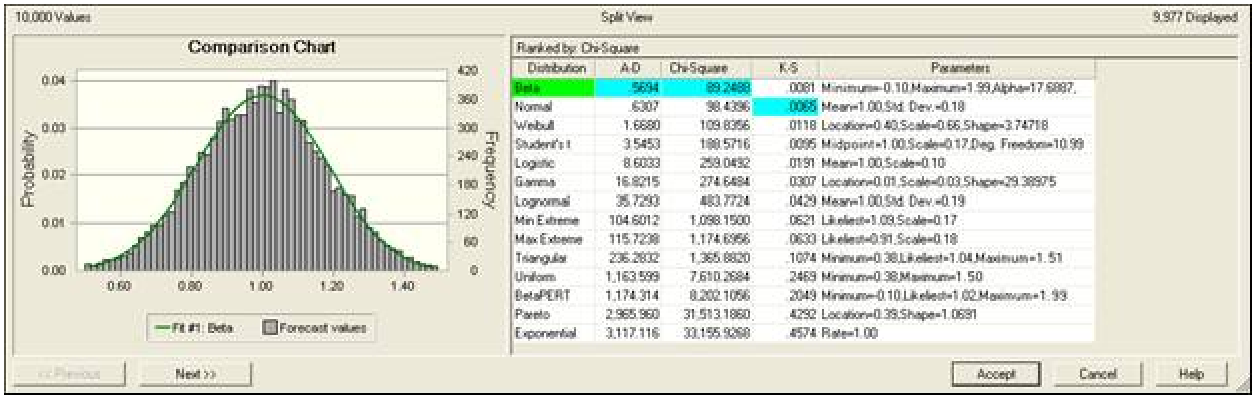


Distribution	A-D	Chi-Square	K-S	Parameters
Triangular	125.5000	1,015.8472	.0473	Minimum=-0.02,Likeliest=1.31,Maximum=2.31
Weibull	6.4502	1,197.4876	.0294	Location=-1.26,Scale=2.65,Shape=5.37272
Beta	8.7487	1,212.6280	.0315	Minimum=-1.14,Maximum=2.53,Alpha=6.61126,Beta=3.81375
Normal	24.3844	1,274.8324	.0356	Mean=1.19,Std. Dev.=0.52
Student's t	25.0996	1,396.2664	.0305	Midpoint=1.19,Scale=0.49,Deg. Freedom=10.99927
Logistic	23.5697	1,456.7392	.0290	Mean=1.20,Scale=0.30
Min Extreme	41.8509	1,747.0560	.0456	Likeliest=1.44,Scale=0.47
Gamma	143.7616	1,963.6836	.0741	Location=-0.57,Scale=0.19,Shape=9.2062
BetaPERT	207.9383	2,659.1356	.0874	Minimum=-1.14,Likeliest=1.43,Maximum=2.53
Max Extreme	208.6628	2,700.2056	.0836	Likeliest=0.92,Scale=0.54
Uniform	477.6751	3,935.0584	.1270	Minimum=0.00,Maximum=2.30
Exponential	2,023.702	10,612.2560	.2682	Rate=0.84
Lognormal	---	---	---	
Pareto	---	---	---	

- Instead, chose to fit conditional distributions and represent pdf as a combination of a discrete probability at zero and an adjusted pdf for positive values.

Causality Example

- Expert G(Conditional, P(causal) = 0.7); Fit Beta distribution to his conditional sample



- G PDF:
  - If $\theta = 0$, $h_g(\theta)$ based on narrow rectangular slice at zero, such that area = 0.3. Does not overlap rest of pdf.
  - For positive θ within the bounds of the Beta distribution, $h_g(\theta)$ equals 0.7 times the output of the Beta pdf at θ.
- G CDF:
  - If $\theta = 0$, $H_g(\theta) = 0.3$
  - For positive θ within the bounds of the Beta distribution, $H_g(\theta)$ = 0.3 + 0.7 times the output of the Beta cdf at θ.

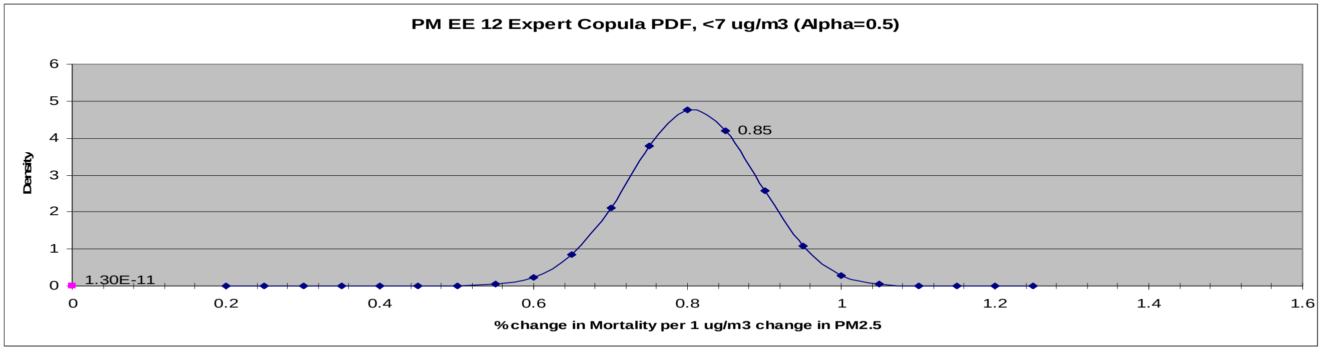
Fitted Expert Distributions

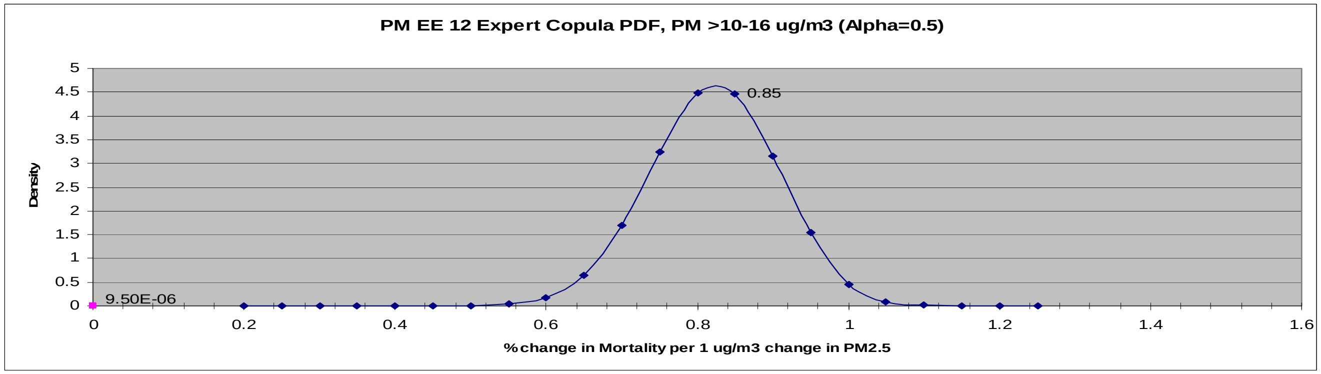
Expert	Distribution	Conditional?	$P(\theta = 0)$	PDF/CDF adjustment
A	Weibull	No	N/A	N/A
B(4-10)	Beta	Yes	0.02	0.98
B(>10-30)	Beta	Yes	0.02	0.98
C	Weibull	No	N/A	N/A
D	Triangular	Yes	0.05	0.95
E	Beta	Yes	0.01	0.99
F(>7-30)	Beta	No	N/A	N/A
F(<7)	Gamma	No	N/A	N/A
G	Beta	Yes	0.3	0.7
H	Beta	No	N/A	N/A
I	Beta	Yes	0.05	0.95
J	Beta	No	N/A	N/A
K(4-16)	Weibull	Yes	0.65	0.35
K(>16-30	Weibull	Yes	0.65	0.35
L(4-10)	Beta	Yes	0.25	0.75
L(<10-30)	Weibull	Yes	0.01	0.99

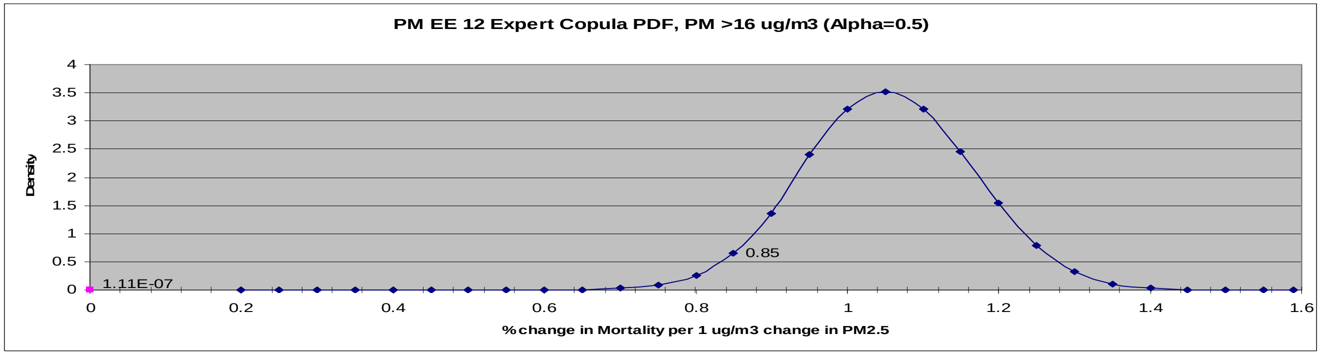
Calculating Fn(θ)

- Developed spreadsheet model to calculate $h_i(\theta)$ and $H_i(\theta)$ for each expert and feed into non-normalized copula function $F(\theta)$.
- Identified θ that maximized $F(\theta)$for a given α; used to select range of θs.
- Calculated $F(\theta)$ for uniformly spaced range of θs.
- "Integrated" resulting curve using trapezoidal approximation and summing areas of each segment to get AUC.
- Normalized $F(\theta)$ by setting k=1/AUC.
- Calculated $F_n(\theta)$ for range of θ's. Result is copula PDF.
- Estimated AUC for $F_n(\theta)$; plotted cumulative AUC for copula CDF.

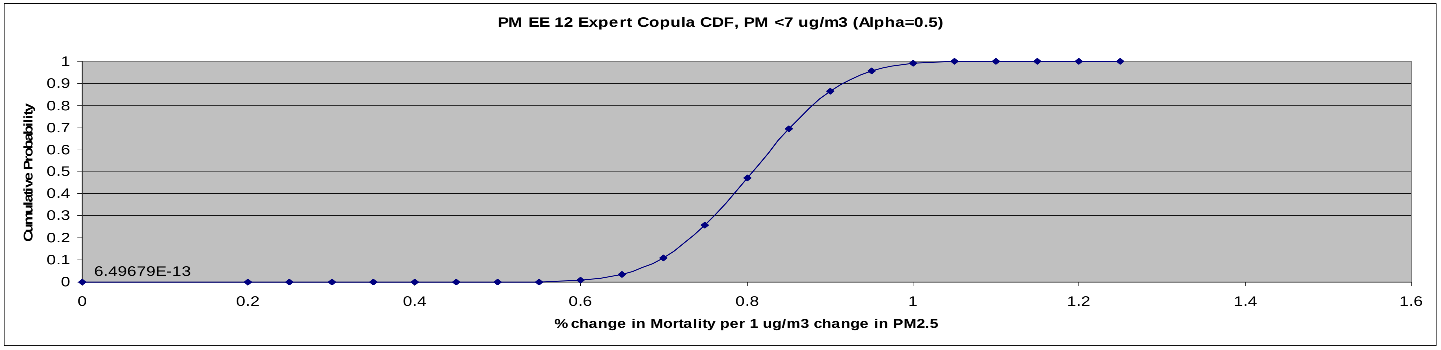
Copula Combined PDFs

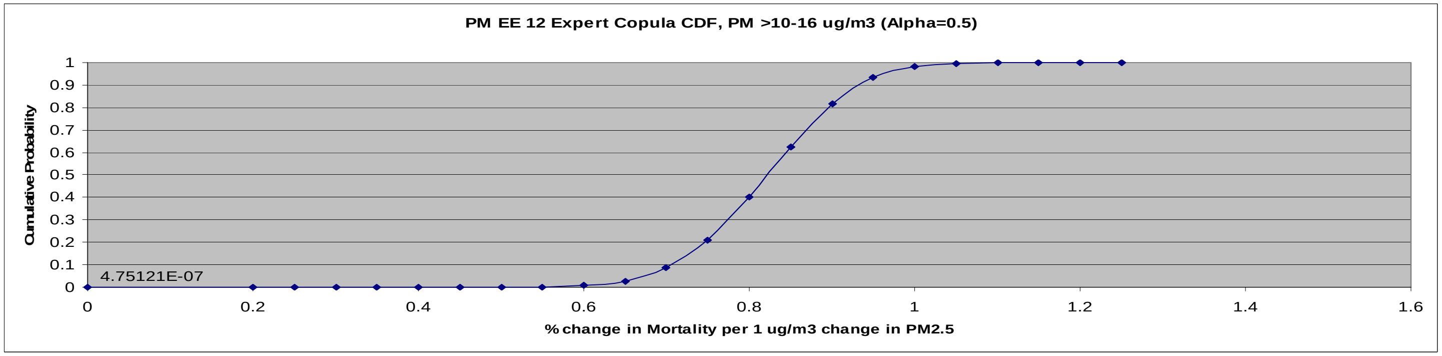


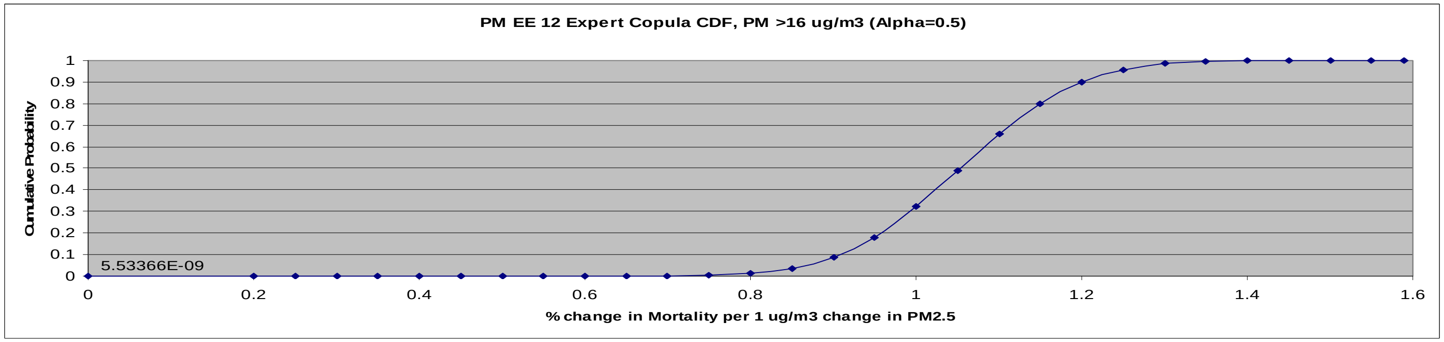




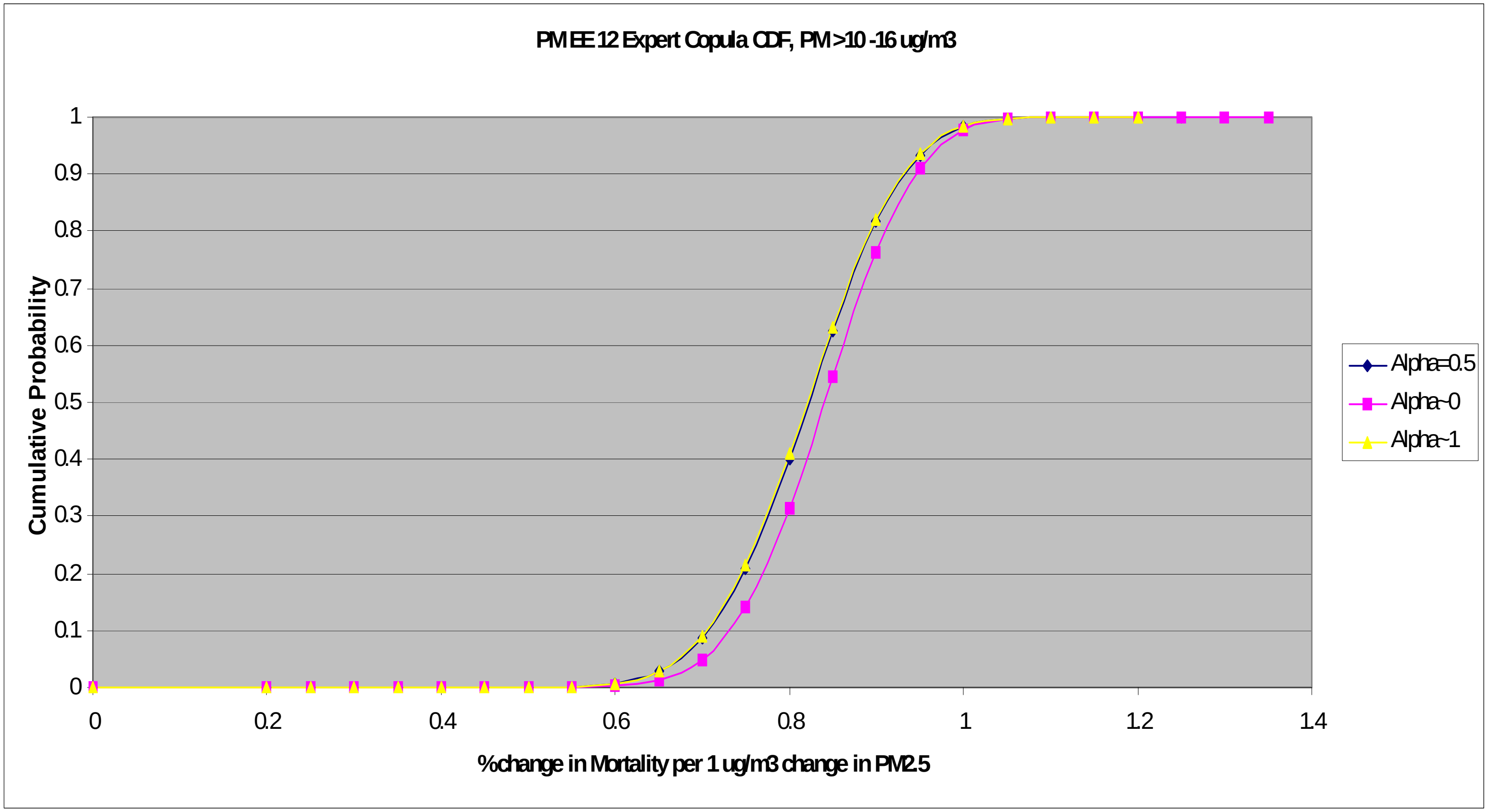
Copula Combined CDFs







Sensitivity Analysis (alpha)



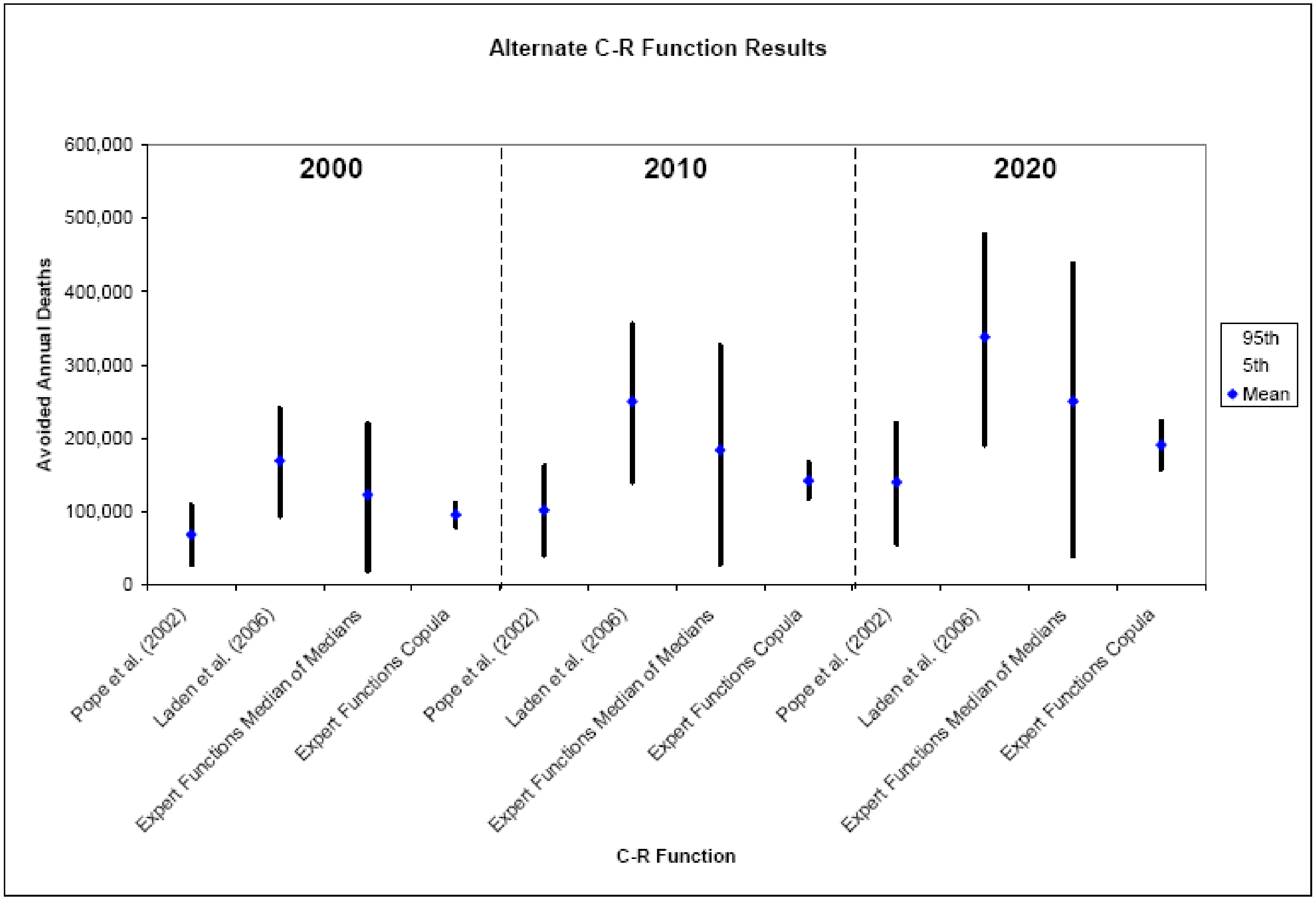
- Results do not appear sensitive to assumptions about dependence.

BenMAP Results

- Copula results for PM C-R coefficient were fed back through Crystal Ball™ to generate a percentile for input into BenMAP.
- Results were pooled across all three PM levels in BenMAP.

C-R FUNCTION	2000			2010			2020		
	PERCENTILE 5	MEAN	PERCENTILE 95	PERCENTILE 5	MEAN	PERCENTILE 95	PERCENTILE 5	MEAN	PERCENTILE 95
Pope et al. (2002)	27,200	68,400	109,000	40,800	102,000	162,000	56,000	140,000	221,000
Laden et al. (2006)	93,800	169,000	241,000	140,000	250,000	356,000	191,000	338,000	478,000
Expert Functions Median of Medians	18,900	123,000	220,000	28,400	184,000	326,000	38,900	250,000	439,000
Expert Functions Copula	79,000	95,700	112,000	118,000	142,000	167,000	158,000	191,000	223,000

Avoided Mortality Comparison



Summary/Next Steps

- Example Copula application produces central estimate of C-R coefficient reasonably consistent with PM EE study results.
- However, produces a dramatically narrower distribution. Different analytical choices may yield alternative results (e.g,. alternative functional forms for the copula, adjustments to tails of distributions to account for potential overconfidence).
- Accounts for dependence, but results evaluated across all 12 experts insensitive to those assumptions. However, some subsets of experts may exhibit greater dependence than the group as a whole.
- Possible next steps include:
  - Copula combinations for subsets of experts
  - Exploring alternative copula specifications

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