

# Lake Champlain BATHTUB Model Calibration Report

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## Contents

**Background**..... 1

**BATHTUB Model Development**..... 2

    Tributary Flow and Load Analysis .....2

    Development of Updated BATHTUB Model Input Datasets .....7

    BATHTUB Input Data .....8

        Lake Morphometry.....8

        Precipitation and Lake Water Levels ..... 10

        Atmospheric Deposition..... 11

        Withdrawals ..... 11

        Point Sources ..... 12

        Outflow 13

        Unmonitored Drainage Area Loads ..... 14

    Updated BATHTUB Model Calibration ..... 16

        Exchange Flow Rate Calibration ..... 16

        Phosphorus Sedimentation Rate Calibration ..... 17

    Chlorophyll-a and Secchi Depth Model Configuration..... 19

        Calibration and Validation Error Statistics ..... 19

    Summary and Conclusions ..... 24

**References** ..... 26

Appendix A. FLUX Results ..... A-1

Appendix B. BATHTUB Input Values, Predicted Flows and Loads and CV Values ..... B-1

Appendix C. BATHTUB Calibration Results ..... C-1

Appendix D. Sensitivity Analysis Plots ..... D-1

**Tables**

Table 1. FLUX predicted discharge volumes (hm<sup>3</sup>/yr) determined at monitoring locations for each of the 21 monitored tributaries, based on current study predicted daily values .....3

Table 2. FLUX predicted discharge volumes (hm<sup>3</sup>/yr) determined at monitoring locations for each of the 21 monitored tributaries, from VTDEC .....4

Table 3. Percent difference in discharge volumes between the current study and VTDEC = (Q<sub>current</sub>-Q<sub>vtdec</sub>)/Q<sub>vtdec</sub>\*100 .....4

Table 4.: FLUX predicted TP loads (mt/yr) determined at monitoring locations for each of the 21 monitored tributaries, based on current study predicted daily values.....5

Table 5. FLUX predicted TP loads (mt/yr) determined at monitoring locations for each of the 21 monitored tributaries, from VTDEC.....5

Table 6. Percent difference in TP loads between the current study and VTDEC = (L<sub>current</sub>-L<sub>vtdec</sub>)/L<sub>vtdec</sub>\*100 .6

Table 7. BATHTUB Input Lake Morphometry.....10

Table 8. Lake water level changes and precipitation depths, 1990-2011 .....10

Table 9. Water Withdrawals from the DFS Study.....11

Table 10. Point Source Model Representation .....12

Table 11. Annual CSO Modeled Parameters.....13

Table 12. Lake Champlain Outflows through Richelieu River for Water Years 1990 through 2011 .....14

Table 13. Calibrated values for exchange flow rates and phosphorus net sedimentation coefficients.....18



Table 14. Chlorophyll-a and Secchi BATHTUB Calibration Factors.....19

Table 15. Percent Error - Calibration and Validation Statistics (Average and Median over all individual segments) parameterized separately for each segment.....20

Table 16. Root Mean Square Error (RMSE) - Calibration and Validation Statistics (Average and Median over all individual segments) parameterized separately for each segment.....20

Table 17. BATHTUB Modeling TCL % Error and R-Squared Statistics .....21

Table 18. TP % Error and R-Squared Statistics .....22

Table 19. TCL Root Mean Square Error Statistics.....23

Table 20. TP Root Mean Square Error Statistics.....23

Table 21. Observed and Final Predicted Lake Segment Average TP and TCL concentrations and CV values for base calibration period (2001-2010).....24

Table 22. Bathtub Predicted Annual Phosphorus Loading by Lake Segment and State/Province (2001-2010 Base Period) .....25

**Figures**

Figure 1. Lake Segmentation Scheme .....9

Figure 2. Unmonitored Drainage Areas (with SWAT-predicted flows and loads) .....15

Figure 3. Calibrated exchange flow rates and cross sectional area at the exchange interface for Lake Champlain segments .....17

Figure 4. BATHTUB Model Calibration Results – Lake Wide Average.....21



## Background

The Vermont Department of Environmental Conservation and the New York Department of Environmental Conservation published the Lake Champlain Phosphorous TMDL jointly in 2002 (VTDEC and NYDEC 2002). This TMDL used a modified version of the U.S. Army Corps of Engineers (COE) steady-state lake water quality model BATHTUB (Walker, 1987, 1996). The basis for the 2002 TMDL analysis was a BATHTUB model developed and calibrated previously, during the Lake Champlain Diagnostic-Feasibility Study (DFS - VTDEC and NYDEC 1997). The DFS BATHTUB model calibration utilized two years of tributary, lake and other monitoring data collected during the period between March 1990 and February 1991 as input. Tributary total phosphorus (TP) and total chloride (CL) load inputs were estimated for the 2-year calibration period using the FLUX program (Walker 1987 and 1996).

The 2002 TMDL BATHTUB model was used to determine watershed point and nonpoint source TP load reductions required to meet in-lake TP criteria established for each of the 13 lake segments. Base case tributary TP loads were estimated for 1991 using tributary flows measured during 1991 and flows and TP loads estimated during the DFS BATHTUB model calibration, for the 2-year period between March 1990 and February 1992.

Based on considerations of phosphorous residence times within the lake and the lack of long-term monitoring data, the TMDL report concluded that the BATHTUB model could not be verified with an independent monitoring dataset, at that time. It also indicated that if permanent losses of phosphorus occur within the tributaries, e.g., due to sedimentation within impoundments, then the relative proportion of tributary point and nonpoint source phosphorous loading assumed during the TMDL would be open to question.

An additional 20 years of tributary, lake and other monitoring data have been collected and compiled by the Lake Champlain Basin Program (LCBP 2010) and much valuable knowledge has been gained by lake researchers, since the original DFS and TMDL BATHTUB modeling. VTDEC (Smeltzer 2009) found that significant (75%) reductions in point source wastewater phosphorus discharges to the lake and its tributaries have occurred, between 1990 and 2008. In recognition of these and other factors, the TMDL process for Lake Champlain is currently being re-visited.

Alternative lake modeling approaches ranging from steady-state models, such as BATHTUB, to time-variable, multi-dimensional models, such as EFDC and WASP, were evaluated by EPA, VTDEP, Tetra Tech, lake stakeholders and researchers, during 2011. As a result, BATHTUB was selected for use as the lake model in the current TMDL analysis (Tetra Tech 2011).

In 2012, the first step in development of the revised TMDL BATHTUB model was completed; namely its update and calibration using the 20 years of tributary, lake and other monitoring data collected since 1992. The updated and calibrated BATHTUB model was subsequently subjected to expert review by Dr. William W. Walker, developer of the BATHTUB application, and in addition, a formal QA review by Dr. Jon Butcher, Watershed Modeling QC Officer for the project. This report describes parameterization of the final calibrated model after incorporation of the findings of both reviews and presents the results.

The BATHTUB model described herein is one of three model and/or analysis tools being applied in the revision of the Lake Champlain TMDL. Each model/tool serves a unique purpose in the TMDL redevelopment process. The BATHTUB model of Lake Champlain is being used to determine whether a specified allocation scenario meets water quality criteria. In addition, SWAT models of the 13 drainage areas contributing flows and phosphorus loads to Lake Champlain are being used to estimate baseline total phosphorus loads from each source sector in each watershed. Finally, a Scenario Evaluation Tool is being used in conjunction with BMP efficiencies to evaluate whether various load reduction scenarios have reasonable potential to meet TMDL loading targets for



Lake Champlain. Reduced loading scenarios from the Scenario Tool are then depicted in the BATHTUB model to test whether a given scenario can meet water quality criteria.

In this context, specific applications of the BATHTUB model in the TMDL revision are as follows:

- Predict current water quality conditions in the 13 lake segments (existing conditions)
- Determine whether test scenarios representative of BMP Implementation scenarios in the basin result in load reductions sufficient to meet water quality criteria (details of that process are outside the scope of this report.)

## BATHTUB Model Development

The BATHTUB model development process was divided into several major efforts, including generation of:

- 1) Tributary nonpoint source and point source flows and water quality parameter loads, using the FLUX32 software and monitoring data collected between 1990 and 2011,
- 2) BATHTUB model input datasets for steady-state simulation of 2-year duration and other time periods between 1990 and 2011, and
- 3) Calibration and validation of the resulting updated BATHTUB models, using lake monitoring data collected between 1990 and 2011.

### Tributary Flow and Load Analysis

The starting point of the model update was development of daily flows and loads of TP, dissolved phosphorus (DP) and Total Chloride (TCL) discharged from the 21 monitored tributaries to the lake. The latest Windows version of the US Army Corps of Engineers FLUX load analysis software (FLUX32) was downloaded and applied to develop these load time series, using tributary flow and water quality data compiled by the Lake Champlain Basin Program's (LCBP) Long-Term Water Quality and Biological Monitoring Program, for the time period between 1990 and 2011.

FLUX was used previously by the Vermont Department of Environmental Conservation (Smeltzer 2009) to estimate TP loading rates at the mouths of the 21 monitored tributaries to the lake, for 2-year periods between 1991 and 2008. The VTDEC analysis, which now extends through 2010, was used as the starting point for the current work. Tributary TP loads developed by Smeltzer are compared to those generated during the current work, later in this report.

The current FLUX loading analysis first sub-divided TP, DP and CL measured at each tributary monitoring location into a maximum of 4 flow strata. FLUX calculation method 6 (best-fit regression to log concentration versus log flow) was then applied, within each flow strata, to determine best-fit regression lines and daily TP, DP and CL loads, for the full time period between 1990 and 2011.

A spreadsheet used during the VTDEC loading analysis to extrapolate flows and FLUX predicted loads at tributary monitoring locations downstream to tributary mouths was obtained from VTDEC. This spreadsheet contains annual-average flows and TP concentrations and loads (calculated from monthly values) discharged from ninety-eight (98) wastewater treatment facilities located within the Lake Champlain Basin, between 1990 and 2011.

Calculation of tributary mouth flows and loads utilized the VTDEC spreadsheet to:

- 1) Subtract point source wastewater loads upstream of tributary monitoring locations,

- 2) Extrapolate the remaining nonpoint source flows and loads to tributary mouths using drainage area ratios<sup>1</sup>, and
- 3) Add point source wastewater loads determined in 1 above back into the extrapolated nonpoint loads at the mouths.

The VTDEC spreadsheet was also used to determine wastewater flows and loads discharged downstream of tributary monitoring locations, i.e. directly to the lake.

Wastewater effluent TCL concentrations are only available for 17 facilities discharging directly to the lake, during the intensive survey period (1990-1991) of the Lake Champlain Diagnostic-Feasibility Study (VTDEC and NYDEC, 1997). For the current work, it was assumed that these effluent TCL concentrations have remained unchanged since the 1990-1991 period. Also, it was assumed that these TCL concentrations are applicable at other facilities within the Lake Champlain Basin, for which no data are available.

Scatter plots contained in Appendix A show monitored flows and FLUX calculated versus observed TP and CL loads on days when water quality data were collected, at each tributary monitoring location. Coefficients of variation (CV=standard error/mean) of the observed loads (flow times concentration) and coefficients of determination (R<sup>2</sup>) for the FLUX determined regression fits at each tributary monitoring location are also contained on these plots.

FLUX predicted tributary water volumes and TP loads estimated during the current study are compared to those developed by VTDEC, in Tables 1 through 6.

**Table 1. FLUX predicted discharge volumes (hm<sup>3</sup>/yr) determined at monitoring locations for each of the 21 monitored tributaries, based on current study predicted daily values**

Trib Name	1991-92	1993-94	1995-96	1997-98	1999-00	2001-02	2003-04	2005-06	2007-08	2009-10
Ausable	649	688	642	890	705	597	673	806	778	647
Bouquet	309	318	288	402	320	245	337	413	383	305
Great Chazy	276	329	297	403	325	263	319	343	333	310
Jewet	1	1	1	2	1	1	1	2	3	3
Lamoille	1208	1135	1168	1376	1182	1121	1296	1358	1461	1154
LaPlatte	40	36	41	51	41	30	45	57	61	41
L. Ausable	43	45	45	89	61	39	48	59	57	42
L. Chazy	44	52	52	76	58	45	54	66	68	46
Lewis	100	83	92	99	96	62	87	117	113	91
L. Otter	56	45	59	64	54	38	57	62	78	58
Mettawee	207	252	221	254	251	102	270	290	298	252
Missisquoi	1333	1351	1517	1735	1519	1399	1764	1786	1975	1544
Otter	977	1102	1129	1281	1146	939	1285	1340	1459	1423
Pike	251	266	239	481	194	164	217	240	239	206
Poultney	226	257	234	239	245	196	296	284	331	269
Putnam	72	66	63	76	64	56	77	94	78	69
Rock	50	53	48	96	39	26	29	34	39	28
Salmon	55	62	52	91	64	50	56	62	67	49
Saranac	783	877	774	1078	875	768	889	981	1027	871
Stevens	5	5	5	9	4	3	4	8	11	2
Winooski	1699	1541	1690	1936	1727	1401	1702	2080	2205	1721
Total Tribs	8329	8505	8604	10622	8926	7516	9471	10437	11009	9097

1 hm<sup>3</sup> = 1,000,000 m<sup>3</sup>

<sup>1</sup> Initial BATHTUB modeling estimated flows, TP, and TCL loads for ungaged areas downstream of monitoring stations by extrapolating flux estimates for adjacent areas and drainage area ratios. In the final calibrated model however, flows and TP loads for these ungaged areas were estimated from SWAT results. Please see 14 for a detailed description.



**Table 2. FLUX predicted discharge volumes (hm<sup>3</sup>/yr) determined at monitoring locations for each of the 21 monitored tributaries, from VTDEC**

Trib Name	1991-92	1993-94	1995-96	1997-98	1999-00	2001-02	2003-04	2005-06	2007-08	2009-10
Ausable	648	689	642	891	705	597	672	806	778	647
Bouquet	309	319	288	402	320	245	336	413	383	305
Great Chazy	276	330	297	403	325	263	319	344	332	310
Jewet										3
Lamoille	1207	1135	1167	1377	1181	1122	1295	1359	1460	1155
LaPlatte	40	36	41	51	41	30	45	57	61	41
L. Ausable	43	45	45	89	61	39	48	59	57	42
L. Chazy	44	52	52	76	58	45	54	66	68	46
Lewis	99	83	92	99	96	62	87	117	113	91
L. Otter	56	45	59	64	54	38	57	62	78	58
Mettawee	207	253	221	254	251	102	270	290	298	252
Missisquoi	1332	1351	1515	1736	1518	1400	1763	1787	1973	1545
Otter	977	1103	1128	1282	1145	939	1089	1340	1458	1424
Pike	251	266	239	498	194	164	217	241	238	207
Poultney	226	257	234	239	245	196	296	284	331	269
Putnam	72	66	63	76	64	56	76	94	78	69
Rock										28
Salmon	55	62	52	91	64	50	56	62	67	49
Saranac	783	878	773	1078	874	769	888	982	1026	871
Stevens										7
Winooski	1697	1542	1688	1937	1726	1402	1701	2081	2202	1723
Total Tribs	8322	8511	8596	10644	8921	7519	9269	10444	11001	9142

1 hm<sup>3</sup> = 1,000,000 m<sup>3</sup>

**Table 3. Percent difference in discharge volumes between the current study and VTDEC = (Qcurrent-Qvtdec)/Qvtdec\*100**

Trib Name	1991-92	1993-94	1995-96	1997-98	1999-00	2001-02	2003-04	2005-06	2007-08	2009-10
Ausable	0.1	-0.1	0.0	-0.1	0.0	0.0	0.1	-0.1	0.0	0.0
Bouquet	0.1	-0.2	0.1	-0.1	0.1	0.1	0.2	0.0	0.1	-0.1
Great Chazy	0.2	-0.2	0.2	0.1	0.0	-0.1	0.0	-0.2	0.2	-0.1
Jewet										5.4
Lamoille	0.1	0.0	0.1	-0.1	0.0	-0.1	0.0	0.0	0.1	-0.1
LaPlatte	-0.4	0.0	0.2	0.0	0.1	1.2	1.0	0.0	-0.6	0.0
L. Ausable	0.5	-0.1	0.1	0.0	0.1	-0.3	0.0	-0.3	-0.5	-0.1
L. Chazy	-1.0	-0.1	0.1	-0.1	0.0	0.9	0.7	-0.2	0.0	-0.1
Lewis	0.6	0.0	0.1	0.0	0.1	0.6	0.5	0.3	0.0	-0.1
L. Otter	-0.2	0.0	0.1	-0.1	0.1	-0.2	-0.8	-0.8	0.1	-0.1
Mettawee	-0.2	-0.2	0.0	0.1	0.0	0.0	0.1	0.0	0.1	-0.1
Missisquoi	0.1	0.0	0.1	-0.1	0.1	-0.1	0.1	-0.1	0.1	-0.1
Otter	0.0	-0.1	0.1	-0.1	0.1	0.0	18.0	0.0	0.1	-0.1
Pike	0.2	0.1	0.1	-3.4	0.2	-0.2	0.1	-0.2	0.3	-0.1
Poultney	0.2	0.1	0.2	0.1	0.0	0.0	0.0	-0.1	0.1	-0.1
Putnam	-0.3	0.0	0.1	-0.1	0.1	-0.2	0.7	-0.2	0.0	0.0
Rock										-1.4
Salmon	0.4	0.0	0.1	-0.1	0.1	0.1	-0.5	0.3	-0.5	-0.1
Saranac	0.0	-0.1	0.1	0.0	0.1	-0.1	0.1	-0.1	0.1	-0.1
Stevens										-70.1
Winooski	0.1	-0.1	0.1	-0.1	0.1	-0.1	0.1	0.0	0.1	-0.1
Total Tribs	0.1	-0.1	0.1	-0.2	0.1	0.0	2.2	-0.1	0.1	-0.5

In general the FLUX predicted discharge volumes determined at monitoring locations for each of the 21 monitored tributaries, during the current study, are very close to those estimated by VTDEC. On a total lake basis, the differences in discharge volumes between the two studies are less than 1%, for all of the 2-year time-averaging periods. Tables 4 and 5 give FLUX TP load results (metric tons per year, mt/yr) for the current study



and those estimated by VTDEC. Table 6 gives the percent difference in TP loads between current study and VTDEC =  $(L_{current} - L_{vtdec}) / L_{vtdec} * 100$ . Monitoring data for Jewett, Rock and Stevens Creeks are not available for the majority of the analysis period. Flow estimates for the unmonitored periods for these drainage areas were derived on the basis of proportional relationships between the Pike Creek watershed flow data and available flow data for these basins. Following augmentation of available flow data record with estimated flow data, Tt applied FLUX to generate loading estimates for these tributary areas for the entire modeling period.

**Table 4.: FLUX predicted TP loads (mt/yr) determined at monitoring locations for each of the 21 monitored tributaries, based on current study predicted daily values**

Trib Name	1991-92	1993-94	1995-96	1997-98	1999-00	2001-02	2003-04	2005-06	2007-08	2009-10
Ausable	23.1	26.7	36.5	62.8	31.9	31.2	26.2	31.4	27.8	29.4
Bouquet	18.5	20.6	18.3	35.8	17.6	13.4	20.7	28.5	24.6	15.9
Great Chazy	16.7	22.6	21.0	36.5	24.8	14.8	19.6	20.4	20.3	19.3
Jewet	0.4	0.4	0.4	0.9	0.3	0.3	0.3	0.7	1.0	1.5
Lamoille	41.4	39.1	47.9	88.2	44.6	43.4	54.3	58.0	58.8	35.8
LaPlatte	8.0	5.4	6.6	8.5	5.7	5.5	7.2	9.0	8.1	5.8
L. Ausable	3.5	3.6	3.2	8.8	4.6	2.5	3.7	4.5	3.4	2.7
L. Chazy	3.7	4.4	4.4	8.2	5.7	3.6	4.7	5.1	4.8	3.5
Lewis	8.1	5.1	7.2	9.3	10.2	5.4	7.8	13.3	9.2	7.5
L. Otter	7.2	5.1	7.7	12.1	6.9	5.1	8.0	8.9	10.3	7.6
Mettawee	15.6	20.7	28.4	27.1	25.4	10.6	24.4	26.7	26.7	20.4
Missisquoi	104.0	98.0	123.0	205.7	105.9	119.7	177.7	175.7	147.9	103.4
Otter	84.7	89.2	92.2	122.1	93.7	71.6	103.7	115.7	119.2	102.2
Pike	41.3	32.0	36.0	85.5	21.8	18.7	30.2	30.2	27.8	23.8
Poultney	19.6	20.1	23.3	21.1	21.5	16.1	27.1	25.4	30.1	24.0
Putnam	1.8	1.7	4.8	1.9	1.5	1.5	1.9	2.5	1.7	2.0
Rock	17.2	18.9	16.5	45.6	10.8	7.4	7.7	10.0	8.7	7.1
Salmon	3.1	2.6	1.9	4.7	2.6	1.7	2.2	2.2	2.3	1.7
Saranac	19.99	25.00	21.54	42.56	23.30	21.24	22.86	26.06	27.30	24.29
Stevens	1.64	1.50	1.50	6.13	0.88	0.62	0.89	5.22	7.64	0.63
Winooski	124.32	88.65	135.86	134.75	142.55	106.25	134.89	179.72	189.11	106.99
Total Tribs	545	511	620	916	590	492	677	763	739	536

**Table 5. FLUX predicted TP loads (mt/yr) determined at monitoring locations for each of the 21 monitored tributaries, from VTDEC**

Trib Name	1991-92	1993-94	1995-96	1997-98	1999-00	2001-02	2003-04	2005-06	2007-08	2009-10
Ausable	25.1	29.6	56.7	45.1	39.5	28.3	27.1	29.1	30.0	35.1
Bouquet	19.5	27.6	24.3	33.4	23.3	11.8	20.7	27.3	24.3	18.2
Great Chazy	16.8	22.7	24.9	32.6	27.7	15.6	23.0	18.8	20.4	23.1
Jewet										1.6
Lamoille	39.6	32.3	35.7	84.4	51.7	46.2	54.7	54.1	57.2	36.9
LaPlatte	10.7	5.7	7.5	7.2	4.3	4.6	6.7	6.5	5.9	4.0
L. Ausable	3.5	4.7	4.7	7.1	7.2	2.8	3.7	4.8	3.2	3.3
L. Chazy	4.5	3.6	5.3	6.9	8.0	4.3	6.1	5.1	4.5	3.4
Lewis	7.7	4.6	6.2	9.6	10.6	5.6	9.0	13.0	8.6	8.6
L. Otter	7.3	5.5	7.4	12.0	7.1	4.6	8.2	8.5	9.4	8.5
Mettawee	15.8	25.2	24.3	35.3	22.4	8.4	24.6	25.1	24.7	18.5
Missisquoi	106.5	109.5	109.9	207.3	125.8	114.1	159.9	164.0	167.0	101.7
Otter	118.9	104.9	88.8	136.0	87.1	58.6	88.7	102.0	120.9	91.9
Pike	59.6	40.2	37.6	69.8	24.2	17.7	26.6	32.6	22.5	22.8
Poultney	21.5	22.6	23.1	26.1	25.3	14.4	27.6	24.9	23.5	32.7
Putnam	2.0	1.6	4.3	2.5	1.6	1.5	2.2	2.1	1.9	2.0
Rock										7.5
Salmon	2.2	3.2	2.0	3.9	3.1	1.9	2.3	2.0	2.2	2.1
Saranac	18.8	25.5	22.8	28.8	24.5	18.8	25.6	26.4	26.3	27.5
Stevens										1.9
Winooski	115.0	108.2	137.1	178.9	151.4	95.7	138.9	179.6	231.2	107.7
Total Tribs	595	577	623	927	645	455	656	726	784	548

**Table 6. Percent difference in TP loads between the current study and VTDEC =  $(L_{current} - L_{vtdec})/L_{vtdec} * 100$** 

Trib Name	1991-92	1993-94	1995-96	1997-98	1999-00	2001-02	2003-04	2005-06	2007-08	2009-10
Ausable	-8	-10	-36	39	-19	10	-3	8	-7	-16
Bouquet	-5	-25	-25	7	-24	13	0	4	1	-12
Great Chazy	0	-1	-16	12	-10	-5	-15	8	0	-16
Jewet										-4
Lamoille	4	21	34	5	-14	-6	-1	7	3	-3
LaPlatte	-25	-5	-12	17	32	19	8	39	37	45
L. Ausable	-1	-24	-32	24	-36	-10	0	-7	5	-17
L. Chazy	-17	22	-16	19	-28	-16	-23	0	8	1
Lewis	5	10	15	-3	-4	-4	-13	3	7	-13
L. Otter	-1	-7	4	0	-2	11	-3	4	10	-11
Mettawee	-1	-18	17	-23	14	27	-1	6	8	10
Missisquoi	-2	-11	12	-1	-16	5	11	7	-11	2
Otter	-29	-15	4	-10	8	22	17	13	-1	11
Pike	-31	-20	-4	23	-10	5	14	-7	23	4
Poultney	-9	-11	1	-19	-15	12	-2	2	28	-27
Putnam	-10	10	13	-23	-6	3	-12	20	-13	-3
Rock										-5
Salmon	40	-18	-8	22	-14	-9	-2	9	4	-21
Saranac	6	-2	-5	48	-5	13	-11	-1	4	-12
Stevens										-67
Winooski	8	-18	-1	-25	-6	11	-3	0	-18	-1
Total Tribs	-8	-12	0	-1	-8	8	3	5	-6	-2

In general, FLUX predicted TP loading rates determined at the monitoring locations for each of the 21 monitored tributaries are similar to those estimated by VTDEC. However, significant differences in TP loading rates as high as 40% were found for individual 2-year periods. On a total lake basis, the differences in TP loads between the two studies are less than 12.1%, for all of the 2-year time-averaging periods. The significant differences found for some tributaries and time averaging periods are likely due to differences in the number of flow strata used during FLUX analysis. VTDEC used 1 or 2 flow strata for regression line development, whereas the current study utilized the maximum of 4 flow strata allowed by the FLUX program. The large differences found for some tributaries and time averaging periods are also likely due to the fact that Smeltzer subdivided the flow and TP observations into 2-year duration periods, prior to flow stratification and regression line development, whereas the current study utilized the full 20+ years of flow and TP data for regression line development.

In summary, significant differences in tributary TP load estimates developed by alternative methods are to be expected, due to the sparseness of the tributary TP monitoring data. Extended gaps in these TP data are particularly evident during the winter months, when tributary flows and TP loads may be elevated, due to large precipitation events. Development and calibration of the physically based, time variable watershed model during the current study will serve to define the relative impacts of watershed TP control strategies on tributary TP loading to the lake BATHTUB model.

Tributary TP loads developed by VTDEC carry considerable weight, since they were developed based on a FLUX analysis of individual 2-year periods (capturing trends over the years) and have been accepted as the most defensible values currently available. Accordingly, the current BATHTUB TP calibration uses the VTDEC tributary TP load values. The physically based, time variable watershed model will also utilize the VTDEC tributary TP load values for calibration.

Since VTDEC did not determine tributary CL loads, these are determined during the current study, using FLUX32. Tributary CL loads to the lake have increased significantly during the 1990s and have stabilized since the early 2000s. The current FLUX32 analysis accounts for these increases in tributary CL loads over the years by subdividing the tributary data into pre and post water year 2001 periods, prior to regression line development.

USGS recently developed tributary TP load estimates using alternative methods. However, the USGS results were not yet available for comparison to those developed during the current BATHTUB modeling and those developed by VTDEC.

## Development of Updated BATHTUB Model Input Datasets

Daily tributary mouth flows and TP, DP and CL loads determined with the FLUX software were subsequently used in a spreadsheet, along with other types of data (e.g., annual rainfall and lake water level variations); to develop BATHTUB input datasets for numerous time-averaging periods. Daily tributary flows and loads were used as the basis for model input dataset development so that daily values predicted with a transient watershed hydrologic and water quality model, such as is currently under development, could also be used directly to develop BATHTUB input datasets.

As the starting point in the current modeling, a BATHTUB input dataset developed previously during the Lake Champlain Phosphorus TMDL (VTDEC and NYDEC 2002) was input to the master spreadsheet as static data. The previous BATHTUB model had been calibrated to lake water quality observations made during the 2-year period, between March 1990 and February 1992. This calibrated BATHTUB model (also referred to here as the DFS-Diagnostic Feasibility Study model) was subsequently applied during the previous TMDL scenario analysis, using tributary and point source flows and TP loads and other inputs developed for calendar year 1991 (the base case existing condition scenario).

During the current modeling, additional sub-sheets were also populated and linked with the master spreadsheet. Linked sub-sheets included:

- 1) **Tribs** - daily tributary mouth flows and TP, DP and CL loads and calculated tables with flows and loads for numerous time-averaging periods,
  - 2) **WWTFs** – annual effluent flows and TP and CL loads and calculated tables with flows and loads for numerous time-averaging periods,
  - 3) **WL&PPT** – annual lake water levels (Rouse’s Point) and rainfall volumes and calculated tables with lake level changes and rainfall volumes for numerous time-averaging periods, and
  - 4) **LakeDat** – lake TP, DP, TCL, chlorophyll-*a* and secchi disc depth data (generated with a FORTRAN program during this work) within each lake segment, for numerous time-averaging periods.
- The resultant spreadsheet contains all the data needed to generate a BATHTUB input dataset, on the fly, for any of the following time-averaging periods:

- 1) March 1990 through February 1992 (DFS Model),
- 2) 2-year duration periods starting on any year between 1991 and 2010 (20 datasets),
- 3) 5-year duration periods starting on 1992, 1997, 2002 and 2007 (4 datasets),
- 4) 10-year duration periods starting on 1992 and 2002 (2 datasets),
- 5) 10-year duration period between 2001 and 2010 (update calibration period), and
- 6) 20-year duration period starting on 1992.

It is important to note that all of these steady-state modeling periods are in terms of Water Years, which start on October 1 of the previous calendar year and extend through September 30 of the given calendar year. The one exception to this is the 2-year duration period of the DFS model, which started on March of 1990 and extended through February of 1992. Based on considerations of the residence time of water and TP within the lake, a minimum 2-year duration time period was determined to be appropriate for simulation with a steady-state model, such as BATHTUB. All available samples in the database for monitoring stations and lake segments were used. Over time there have been some changes in monitoring locations and sampling frequencies. Because BATHTUB



averages together an entire lake segment, changes in sampling frequency and monitoring locations changes are averaged out.

The BATHTUB input dataset generation spreadsheet (MAKBATHTUBInputs.xlsx) was subsequently tested and used to generate all the datasets needed during the model testing and calibration phase of the current work. The predicted daily tributary flows and loads from the watershed can be directly input to this spreadsheet and BATHTUB input datasets can be generated to allow simulation of alternative management and climate change scenarios with the calibrated BATHTUB model. Two versions of this spreadsheet were developed so that either predicted daily tributary TP values from a watershed model or FLUX analysis, or those published by VTDEC, may be used. Ultimately, FLUX-predicted values were used.

It is important to note that the resultant BATHTUB input datasets contain not only the tributary flows, loads and all other information (e.g., lake segment geometry and segment connectivity), but also the spatially averaged (within segments) and temporally averaged lake water quality observations, corresponding to each simulation time period. The uncertainty/error statistic (CV) of each model input variable and the observed lake data are also contained in each input dataset.. These CV values were used as input to the BATHTUB model of the current study. All baseline (2001-2010 average) flows, loads (and/or flow-weighted concentrations), and CV values used as inputs to calibrate the model are provided in Appendix B.

BATHTUB output includes values of CV (and hence the standard error) for each output water quality variable, within each reach, based on a first-order analysis of error due to the combination of modeling inputs. BATHTUB output also includes values of CV for the lake observations, so that calibration plots can include standard error bars for both predicted and observed values.

## BATHTUB Input Data

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The numerous types of input data contained in the BATHTUB dataset generation spreadsheet are presented and discussed below. Specifically the following input data are discussed: lake morphometry, precipitation, lake level, atmospheric deposition rates, water withdrawals, point sources, lake outflow, and estimates for unmonitored drainage areas.

### Lake Morphometry

Lake Segment geometric characteristics (morphometry) contained in the 2002 TMDL BATHTUB model input dataset were used, without modification, in the current BATHTUB modeling. Figure 1 shows the BATHTUB segmentation of the lake; Table 7 gives the surface area, mean water depth (total and mixed), length, volume and width specified for each BATHTUB lake segment. It also lists the downstream segment number receiving outflows from each lake segment. Segment 9 (Mallet's Bay) has two outlets: one discharging to segment 5 (Main Lake) and an additional channel discharging to segment 10 (Northeast Arm). During the DFS study it was estimated that 19 percent of the total tributary inflow to Mallet's Bay is discharged through this channel to the Northeast Arm. The current BATHTUB model also uses this fraction.

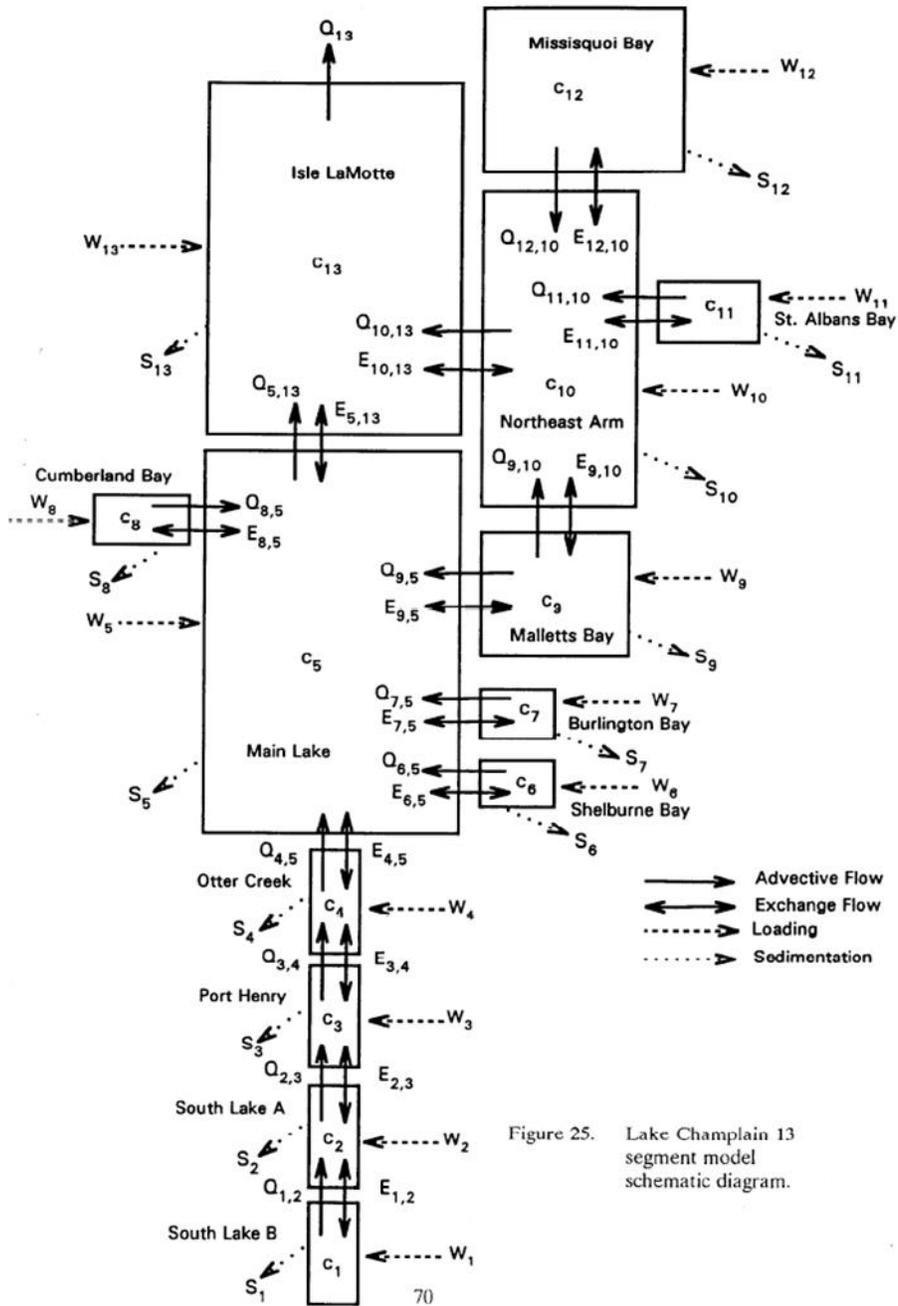


Figure 1. Lake Segmentation Scheme



Table 7. BATHTUB Input Lake Morphometry

Segment No. and Name	Out Segment	Area (km <sup>2</sup> )	Mean Depth (m)	Mixed Layer Depth (m)	Length (km)	Volume (hm <sup>3</sup> )	Width (km)
1 South Lake B	2	5.8	1.4	1.4	20.1	8.1	0.3
2 South Lake A	3	43.3	2.9	2.9	33.5	125.6	1.3
3 Port Henry	4	75.6	19.4	8.2	20.1	1,466.6	3.8
4 Otter Creek	5	28.5	33.5	8.3	10.1	954.8	2.8
5 Main Lake	13	414.1	40.5	8.2	47.0	16,771.1	8.8
6 Shelburne Bay	5	9.6	14.6	7.7	3.4	140.2	2.8
7 Burlington Bay	5	5.5	11.4	7.1	2.0	62.7	2.8
8 Cumberland Bay	5	10.6	5.9	5.1	3.4	62.5	3.1
9 Mallett's Bay	5	55.1	13.1	7.5	6.7	721.8	8.2
10 Northeast Arm	13	248.3	13.6	7.5	33.5	3,376.9	7.4
11 St Albans Bay	10	7.2	3.2	3.2	3.4	23.0	2.1
12 Missisquoi Bay	10	89.9	2.3	2.3	16.8	206.8	5.4
13 Isle La Motte		185.6	10.2	6.8	40.3	1893.1	4.6
Totals		1,179.1				25,813.1	

1 hm<sup>3</sup> = 1,000,000 m<sup>3</sup>

### Precipitation and Lake Water Levels

Precipitation data are needed by the BATHTUB model to determine the average volumetric rate of rainwater deposition directly onto individual lake segments, during each simulation time period. These input rates (Table 8) were determined using monthly summaries of hourly precipitation data collected by the National Oceanic and Atmospheric Administration (NOAA) at the Burlington Airport, Vermont (WBAN 14742), between 1990 and 2011. In addition the U.S. Geological Survey (USGS) daily water levels measured at Rouse's Point were used to determine changes in water year averaged lake water levels between 1990 and 2011 for Lake Champlain. These values were used in the BATHTUB dataset generation spreadsheet to calculate water level changes (Table 8) over the various time periods of the BATHTUB model simulations.

Table 8. Lake water level changes and precipitation depths, 1990-2011

Water Year	Delta Level (m)	Rainfall (m)	Water Year	Delta Level (m)	Rainfall (m)
1990		0.90	2001	-1.01	0.68
1991	-0.31	0.97	2002	-0.28	0.86
1992	-1.00	0.79	2003	0.47	0.74
1993	0.51	0.86	2004	1.23	1.18
1994	0.24	0.92	2005	-0.77	0.84
1995	-1.20	0.66	2006	1.38	1.20
1996	1.83	1.07	2007	-0.78	0.96
1997	-0.39	0.79	2008	0.25	1.16
1998	0.50	1.36	2009	-0.62	0.97

Water Year	Delta Level (m)	Rainfall (m)	Water Year	Delta Level (m)	Rainfall (m)
1999	-1.57	0.77	2010	-0.28	0.93
2000	1.21	0.95	2011	1.68	1.44

## Atmospheric Deposition

Average atmospheric deposition rates of TP and CL mass onto the lake surface were calculated during the DFS BATHTUB modeling study, based on sampling of precipitation quality at eight stations, during the 2-year period between March 1990 and February 1992. These deposition rates, which were found to be 16.1 and 0.276 mg/m<sup>2</sup>/yr for TP and CL, respectively, were assumed to be constant over time in the current BATHTUB modeling.

An average lake surface water evaporation rate was determined during the DFS study, based on adjusted pan evaporation rates measured at a NOAA meteorological station located at Essex Junction, Vermont. The DFS evaporation rate of 0.71 meters per year was assumed to be constant over time in the current BATHTUB modeling.

## Withdrawals

Municipal, industrial and private water users withdraw water from Lake Champlain at numerous locations. Table 9 below gives withdrawal rates determined in the DFS BATHTUB modeling study, for the six largest users of lake water. Water withdrawals from the other smaller entities were assumed to be insignificant for modeling purposes.

**Table 9. Water Withdrawals from the DFS Study**

Entity Name	Withdrawal Rate (hm <sup>3</sup> /yr)	Segment Number
International Paper Co., NY	22.2 <sup>a</sup>	2
Champlain Water District, VT	11.5	5
City of Burlington, VT	7.6	5
Village of Swanton, VT	1.1	12
City of St. Albans, VT	1.0	10
Tri-Town Water District, VT	0.8	3
Weed Fish Culture Facility, VT?	8.79 <sup>a</sup>	5

a. Value used for the 2001-2010 model. Flow values vary for other years/models  
1 hm<sup>3</sup> = 1,000,000 m<sup>3</sup>

The Weed Fish Culture Facility withdrawal was not in the DFS model but was included after 1995 onwards when data were available. For purposes of the current BATHTUB modeling, the withdrawal rates given in Table 9 for all entities except the International Paper Company (withdrawal ranges from 21.05 to 24.12 hm<sup>3</sup>/yr) were assumed to be constant over time at DFS values. The withdrawal rates for the Weed Fish Culture Facility also varied with withdrawals ranging from 5.34 to 10.63 hm<sup>3</sup>/yr. VTDEC (Smeltzer 2009) previously determined water year average volumetric discharge rates and TP loading rates for the International Paper Company and the Weed Fish Culture Station facilities, during the period between 1990 and 2011. In the current BATHTUB



modeling, it was assumed that these two facilities withdraw water from the lake at the same rate as they discharge point source effluent back into the lake. All the withdrawals were given a constant CL, TP, and Ortho P concentration of 0.01 mg/L, 0.01 ug/L, and 4 ug/L respectively for all the years for which models were developed.

## Point Sources

### Wastewater Treatment Plants

There were a total of 24 point sources specified in the BATHTUB model that discharged direct to the lake or downstream of the tributary monitoring stations. Table 10 lists the point sources represented in the BATHTUB model and the segment to which they discharge. The associated flow and TP loading for the year 2001 to 2010 as specified in the BATHTUB model are also given in Table 10.

**Table 10. Point Source Model Representation**

WWTP	Flow (hm <sup>3</sup> /yr)	TP (kg/yr)	Lake Segment
Champlain Park	0.14	405.5	8
Crown Point	0.05	144.6	2
International Paper	22.21	4369.5	2
Peru-Valcour	0.01	22.1	5
Plattsburgh	7.90	8936.2	8
Port Henry	0.70	1329.3	3
Rouses Point	0.99	1662.0	13
Ticonderoga	1.63	2095.6	2
Westport	0.18	336.7	3
Wyeth Research	0.06	66.6	13
Venise-en Quebec	0.001	0.0	12
Alburg	0.07	2.7	13
Brown Ledge Camp	0.002	4.0	9
Burlington Main	6.04	2833.6	7
Burlington North	1.57	588.1	5
Northwest State Correction	0.04	4.8	11
Orwell	0.03	82.8	2
Shelburne No1	0.46	170.4	6
Shelburne No2	0.47	180.8	6
S. Burlington Bart. Bay	0.81	191.2	6
St. Albans City	0.81	191.2	11
Swanton	0.67	335.5	12
Vergennes	0.53	182.8	4
Weed Fish Culture Station	8.79	308.8	5

Note: Flow and TP loads are from the 2001-2010 BATHTUB model.

1 hm<sup>3</sup> = 1,000,000 m<sup>3</sup>

### CSO

In addition to the WWTPs, the Burlington CSO was also represented in the BATHTUB model as a point source (going into segment 7). Monthly discharge, rainfall and average TP for the Burlington Combined Sewer Overflow were available for the period 2001 to 2013 (provided by the Burlington Public Works Department) and were used to characterize the CSO for the model. For the portion of the modeling period for which data were not available (1991-2000), CSO values were extrapolated based on a relationship between CSO loads and precipitation.

Specifically relationships between existing Flow vs Rainfall and Flow vs TP loading were derived from the CSO data (both showed a fairly strong R2 value of 0.73 and 0.76 respectively). Table 11 presents the annual loads used to characterize the CSO in BATHTUB. It should be noted that the CSO loads were added explicitly as a point source to segment 7 and that the unmonitored loads for segment 7 were derived from SWAT estimates for the area not draining into the CSO.

**Table 11. Annual CSO Modeled Parameters**

Water Year	Rain (in)	Flow (MG)	Flow (hm <sup>3</sup> )	TP (lbs)	TP (kg)
1990	50.65	223.52	0.85	2,311.11	1,048.30
1991	41.55	183.12	0.69	1,897.67	860.77
1992	39.33	173.28	0.66	1,796.99	815.10
1993	37.00	162.94	0.62	1,691.12	767.08
1994	49.56	218.66	0.83	2,261.40	1,025.75
1995	29.29	128.71	0.49	1,340.80	608.18
1996	54.22	239.37	0.91	2,473.31	1,121.87
1997	41.82	184.31	0.70	1,909.83	866.28
1998	48.73	214.97	0.81	2,223.67	1,008.64
1999	38.02	167.43	0.63	1,737.08	787.93
2000	43.75	192.87	0.73	1,997.45	906.03
2001	10.36	83.31	0.32	1,546.92	701.67
2002	35.54	125.07	0.47	1,517.47	688.31
2003	45.13	83.33	0.32	1,054.85	478.47
2004	47.35	209.64	0.79	1,773.82	804.59
2005	38.60	131.98	0.50	1,330.07	603.31
2006	54.02	222.39	0.84	2,304.70	1,045.39
2007	46.46	193.78	0.73	2,867.69	1,300.76
2008	55.65	178.80	0.68	1,703.77	772.82
2009	43.46	153.48	0.58	1,739.96	789.23
2010	40.68	181.46	0.69	2,655.86	1,204.68

1 hm<sup>3</sup> = 1,000,000 m<sup>3</sup>

## Outflow

The outlet of Lake Champlain, which is near Rouse's Point, discharges through the Richelieu River and eventually flows into the St. Lawrence River, in Quebec, Canada. Daily average flows measured on the Richelieu River at Fryers Rapids (Environment Canada Station 02OJ007) were downloaded from the site:

[www.wateroffice.ec.gc.ca](http://www.wateroffice.ec.gc.ca) and processed for input to the BATHTUB dataset generation spreadsheet.

Fryers Rapids is located some distance downstream on the Richelieu River and flows measured at this station were adjusted to the lake outlet, using their drainage areas (square kilometers) to calculate a ratio equal to 0.974 (Arouses/Afryers = 21437/22000).



Table 12 gives the resultant lake outflow volumes for each water year between 1990 and 2011. These annual values are used in the BATHTUB dataset generation spreadsheet to calculate outflow volumes for the various time periods.

**Table 12. Lake Champlain Outflows through Richelieu River for Water Years 1990 through 2011**

Water Year	hm <sup>3</sup> /yr	Water Year	hm <sup>3</sup> /yr
1990	13580	2001	9723
1991	13364	2002	9322
1992	9317	2003	9794
1993	11438	2004	14498
1994	12235	2005	11717
1995	7850	2006	16845
1996	15343	2007	13872
1997	12847	2008	14995
1998	15413	2009	12359
1999	8789	2010	11686
2000	14202	2011	20008

1 hm<sup>3</sup> = 1,000,000 m<sup>3</sup>

### Unmonitored Drainage Area Loads

Un-monitored watersheds constitute between 1 and 23 percent of the total tributary area discharging to an individual lake segment. Smeltzer (2010) estimated water volumes and TP loads discharged from each un-monitored watershed, using unit area FLUX results for nearby monitored tributaries and un-monitored watershed areas. This method was used in the 2012 calibrated version of the Lake Champlain BATHTUB model. However, it was found that the land use distribution from the nearby monitored tributaries was different from that in the un-monitored watershed areas which could lead to discrepancies in the loading estimates due to extrapolation. In the current model these previous load estimates have been replaced with simulated flow and total phosphorus (TP) loading from the calibrated SWAT watershed model which better represented the land use distribution for the same areas.

SWAT direct drainage loads for each year from 1990 to 2010 (by water year) were summarized for the unmonitored tributary input in each of the 13 segments that were updated. The SWAT TP concentration was calculated using the SWAT generated flow and TP loading. Ortho-P was calculated using the ratio of PO<sub>4</sub>/TP (calculated for each segment using PO<sub>4</sub> and TP using the previously extrapolated values) and multiplying it with the SWAT TP concentration to estimate the revised PO<sub>4</sub> concentrations. The TCL concentrations and associated CV values for flow, TP and TCL were left unchanged from the previously derived estimates from Smeltzer (2010). Concentrations were derived for each modeled year (1991 to 2010) based on the SWAT flows and loads provided. From the yearly derived concentrations and flows average flows and concentrations were calculated for each model period (91-92, 93-94,...2001-2010, etc). Figure 2 shows the unmonitored areas where SWAT predicted flows and loads were used as inputs to the BATHTUB model.

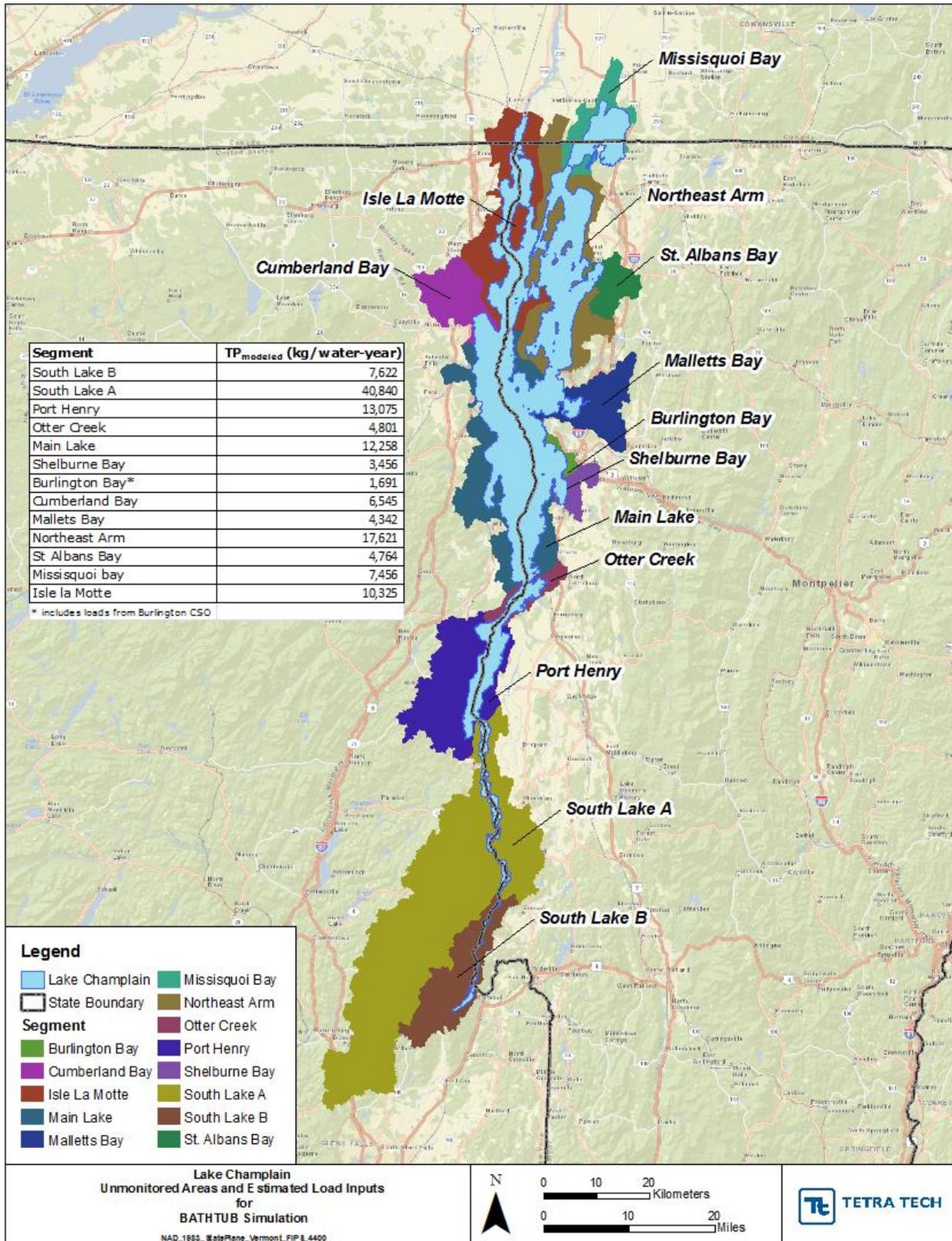


Figure 2. Unmonitored Drainage Areas (with SWAT-predicted flows and loads)

## Updated BATHTUB Model Calibration

The numerous BATHTUB input datasets generated were subsequently used to calibrate the model to the extensive database of lake water quality observations. Tributary TP loads and corresponding CV values published by VTDEC (see Table 5 and Appendix D, respectively) were used exclusively. Flows and loads discharged from un-monitored portions of the tributary watershed for each lake segment were developed based on simulated flow and total phosphorus (TP) loading from the calibrated SWAT watershed model. The resultant flows and loads discharged from each of the un-monitored watershed areas, during each of the 2-year periods, are also given in Appendix D.

The model calibration strategy included calibrating the model to the pooled data for water years 2001-2010 and evaluating against 2-year spans within that range. The model was then also applied to 2-year intervals from 1991-2000, which is essentially a validation test.

### Exchange Flow Rate Calibration

The first step in the model calibration process was to utilize the lake TCL observations within each segment to calibrate longitudinal mixing (dispersion) rates, as TCL can be assumed to be conservative within the lake. The 10-year duration time period between 2001 and 2010 was chosen for calibration of mixing, to capture current conditions.

The BATHTUB model, which is a simple steady-state spatially segmented model, requires assignment of diffusive transport rates between segments of the lake. Initially diffusive exchange was evaluated using the Fischer Equation coupled with segment-by-segment calibration factors. Optimization of the diffusion factors to match observed chloride data yielded extremely variable results even when the factors were constrained within the range of 0.01 to 10.0 (varied over 3-orders of magnitude). The extreme variability in coefficients was interpreted to suggest that the Fischer approach was not appropriate for portions of this lake as segmented. Hence, the exchange rates between lake segments were evaluated using a direct mass balance for chloride.

Exchange flows between lake segments were calculated by constructing a mass balance for chloride between adjacent segment pairs using the average observed TCL data from 2001 to 2010 and directly solving the linear mass balance equations for each segment for the exchange flow terms. The calculated exchange values were quite sensitive to different inputs. One of the biggest reasons for the variability in exchange rates is the small differences in chloride concentrations between adjoining lake segments (which are represented in BATHTUB as control volume boxes of varying sizes, which only have one monitoring location within each) and the sensitivity of the calculated exchange rates to these concentration gradients. In addition, the calculated exchange rates for three of the segments (South Lake B, Mallets Bay, and Missisquoi Bay) yielded negative exchange rates. For these three segments the calculated exchange rates from the DFS model, which was based on 1990 to 1992 data were substituted. This was considered reasonable since the relationship of the 2001 to 2010 calculated exchange rates with the areas of exchange interface is more consistent with the DFS relationship than the calculated exchange rates using the Fischer model (Figure 3).

In addition, it should be noted that the chloride concentrations were not at steady state in the lake between 1990 to 2010, with some significant increases occurring along the main stem of the lake. Of all the approaches evaluated, the calibration of the BATHTUB model using segment-by-segment direct estimation of the exchange rates provided the most direct approach and gave the best calibration, measured in terms of mean percent error and RMSE. The relationship between the calibrated exchange rates and the cross-sectional profile areas of the corresponding interfaces is shown in Figure 3. The 2001 to 2010 relationship indicates that the chloride mass balance method of calculating exchange rates will produce realistic hydrodynamic values for Lake Champlain. This is especially important in Lake Champlain since the segment interfaces include a broad range of

hydrodynamic environments ranging from narrow channel like sections where exchange flows are constricted, to wide and deep open water situations where extensive mixing would be expected.

Despite uncertainty in estimating the exchange rates, this does not appear to be a significant source of uncertainty relative to the seasonally averaged nutrient concentrations that are the key management output of the model. Application of the calibrated exchange rates to the validation period resulted in a mean percent error for chlorides of less than 5% on a total lake basis. The spatial variability of chloride and TP concentrations in the lake is more strongly determined by advective fluxes relative to the locations of point source inputs than by diffusive exchanges between segments.

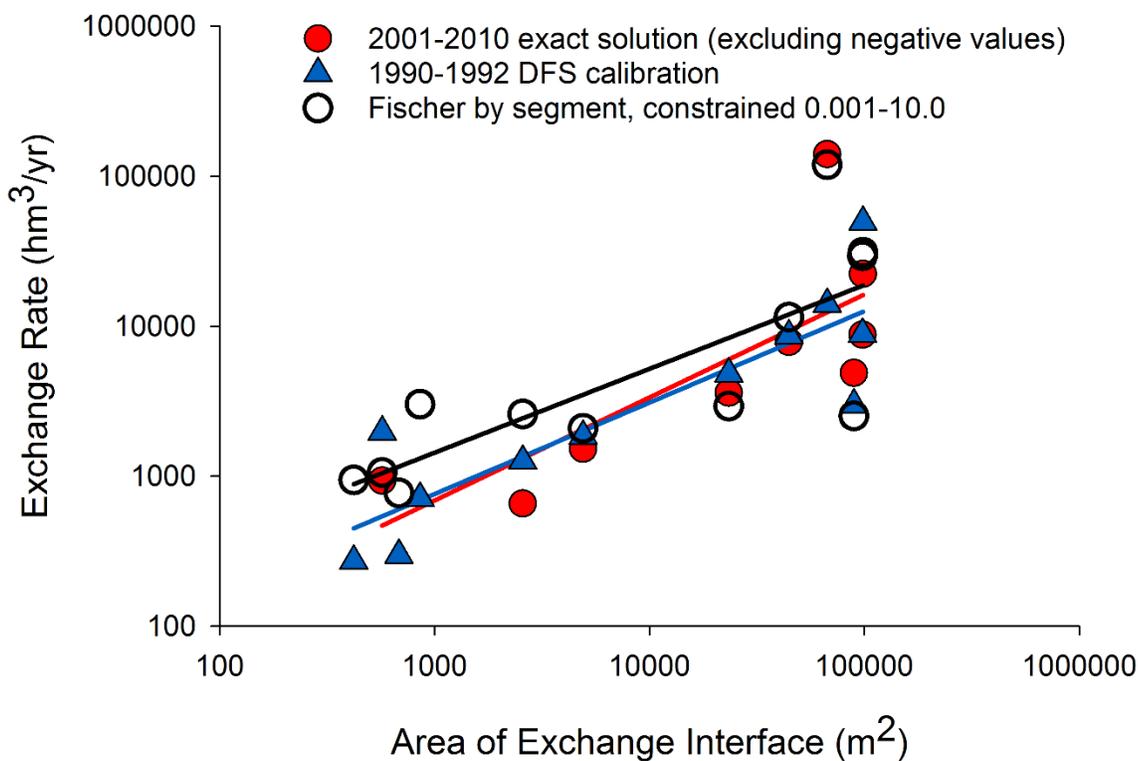


Figure 3. Calibrated exchange flow rates and cross sectional area at the exchange interface for Lake Champlain segments

### Phosphorus Sedimentation Rate Calibration

The next step in the BATHTUB modeling was to calibrate TP loss rates, due to settling of the particulate phosphorus component, within each lake segment. In order to calibrate the model to current conditions, the 10-year duration time period between 2001 and 2010 was also chosen. Accordingly, the calibrated exchange rates determined after the exchange flow rate calibration were first set in the 10-year BATHTUB input dataset starting in 2001.

Several optional methods are available in BATHTUB for simulation of TP settling within each lake segment. The calibrated BATHTUB model of the current study uses Method 3 for predicting phosphorous settling rates. Model 3 is well suited for lakes with complex morphometry such as Lake Champlain and simulates TP settling from the



water column as a second-order process, not accounting for the fraction of each segment's tributary TP loading that is dissolved (DP). Method 3 was also used during the 2002 TMDL BATHTUB modeling. Another optional method (Method 2) for simulating phosphorus settling in BATHTUB includes the impact of the TP/DP ratio, on sedimentation rates, in order to account for differential settling of particle-bound phosphorus. This option was tested and did not result in an improvement in the BATHTUB calibration results.

A segment specific calibration of the sedimentation coefficients was conducted using mass balance to determine the TP settling rate, using Walker's (1987) best fit value of  $100 \text{ m}^3/\text{g-yr}$  (or a TP calibration factor of 1 for Method 3 that was used) from the national reservoir data set as a starting point for each lake. The goal was to bring the predicted concentrations as close as possible to the observed values in all segments without excessively tuning the model. Finally a sedimentation rate of  $140 \text{ m}^3/\text{g-yr}$  was arrived at for 10 of the 13 lake segments using a least squares process. Note that this rate departs from the default BATHTUB term by a factor of 1.4. The other three segments - South Lake B, Malletts Bay, Missisquoi Bay had higher sedimentation rates  $1,200 \text{ m}^3/\text{g-yr}$ ,  $460 \text{ m}^3/\text{g-yr}$ , and  $190 \text{ m}^3/\text{g-yr}$  respectively. These higher rates might be rationalized by describing these three segments as "inflow-dominated" in the sense that they have large rivers entering relatively small or confined areas where sedimentation might be expected to be higher than for the more open and interconnected segments. In addition these rates held up during the validation period and gave reasonably accurate predictions during the validation period justifying the deviation from the other model segments and the default suggested values. Table 13 shows the calibrated values for exchange flow rates and phosphorus net sedimentation coefficients.

**Table 13. Calibrated values for exchange flow rates and phosphorus net sedimentation coefficients**

Segment	Group	Name	Exchange Flow Rate (E) ( $\text{hm}^3/\text{yr}$ )	TP Calibration Factor	Sedimentation Rate ( $\text{m}^3/\text{g-yr}$ )
1	1	South Lake B	712 <sup>a</sup>	12.00	1200
2	2	South Lake A	656	1.40	140
3	2	Port Henry	140,044	1.40	140
4	2	Otter Creek	22,272	1.40	140
5	2	Main Lake	8,762	1.40	140
6	2	Shelburne Bay	3,586	1.40	140
7	2	Burlington Bay	4,885	1.40	140
8	2	Cumberland Bay	7,840	1.40	140
9	3	Mallett's Bay	272 <sup>a</sup>	4.60	460
10	2	Northeast Arm	930	1.40	140
11	2	St Albans Bay	1,519	1.40	140
12	4	Missisquoi Bay	297 <sup>a</sup>	1.90	190
13	2	Isle La Motte	0	1.40	140

<sup>a</sup>Input constrained to DFS predicted E for  $E < 0$   
 $1 \text{ hm}^3 = 1,000,000 \text{ m}^3$

St Albans Bay (segment 11) was a special case where net internal loading from historically enriched lake sediments is known to occur. For St. Albans Bay, during calibration the TP settling rate was set similar to the other segments to a value of  $140 \text{ m}^3/\text{g-yr}$  (or TP factor of 1.4) and in order to fit the observed phosphorus concentration in the bay the internal loading was fine tuned. The resulting TP internal loading for St. Albans Bay was fine tuned to  $4.2 \text{ mg}/\text{m}^2/\text{d}$ . [Cornwell and Owens \(1999\)](#) have reported sediment core flux measurements at two stations in St. Albans Bay. The range was  $0.0$  to  $6.4 \text{ mg}/\text{m}^2/\text{d}$ , with the highest rates observed during summer. The mass balance derived internal load of  $4.2 \text{ mg}/\text{m}^2/\text{d}$  is within the observed range.

## Chlorophyll-a and Secchi Depth Model Configuration

Several optional methods are available in BATHTUB for prediction of algal chlorophyll-a levels within each lake segment. Algal chlorophyll-a was not included during the 2002 TMDL BATHTUB modeling. Method 4 calculates water column chlorophyll-a within each lake segment, using a linear response to predicted water column phosphorous levels. The calibrated BATHTUB model of the current study uses Method 4 for predicting chlorophyll-a levels. Method 4 has applicability constraints of non-algal turbidity (<0.9), and phosphorus limitation ((N-150)/P>12, and IN/PO4 >7), and flushing rate to be less than 25 (which were all verified to be applicable using model diagnostics and observed data). Ratios using phosphorus and nitrogen data indicated that growth is indeed nutrient limited (phosphorus limited). In addition light is not significantly limiting as confirmed by the principal component 2 of trophic response variable (ANTILOG PC-2) which was greater than 10, and the non-algal turbidity values which were around 0.1.

Several optional methods are available in BATHTUB for prediction of secchi depth (light extinction rates) within each lake segment. Method 1 determines secchi depth based on both predicted algae chlorophyll-a levels and specified non-algal (background) turbidity levels (contained in Table 15). The calibrated BATHTUB model of the current study uses method 1 for predicting secchi depth within the water column. Secchi depth was not included during the 2002 TMDL BATHTUB modeling.

**Table 14. Chlorophyll-a and Secchi BATHTUB Calibration Factors**

Segment	Chlorophyll-a	Turbidity
1	0.60	1.00
2	0.60	1.00
3	1.00	1.00
4	1.00	1.00
5	1.00	1.00
6	1.00	1.00
7	1.00	1.00
8	1.00	1.00
9	1.00	1.00
10	1.00	1.00
11	1.00	1.00
12	1.00	1.00
13	1.00	1.00

## Calibration and Validation Error Statistics

The next step was to update the diffusion factors and TP settling rate factors in the BATHTUB input dataset generation spreadsheet (MAKBATHTUBInputs.xlsx) with the calibrated values determined using the 2001 to 2010 average model above to create BATHTUB input datasets. Specifically the following series of 2-year BATHTUB input datasets were updated: DFS (March 1990 through February 1992) and eleven 2-year periods starting in 1991, 1993, 1995, 1997, 1999, 2001, 2003, 2005, 2007, 2009 and 2010. BATHTUB was subsequently run, using each of these datasets as input.

Overall calibration and validation statistics are shown in Table 15 and Table 16. Note that these are summaries over the 2-year results for all individual segments, not the whole-lake statistics). Table 15 shows the percent error which was computed as  $(P - O)/O$ , and Table 16 shows the Root Mean Square Error (RMSE) computed as

$$\sqrt{\sum(P - O)^2/n}$$

As would be expected, the quality of fit declines somewhat when the model is extended to a different time period, as shown by the decrease in  $R^2$ . For total chloride and chlorophyll a the percent error magnitudes remain small



during the validation period; however, for total phosphorus there is a increase in the total phosphorus error during the validation period. The increase in the validation statistics can be attributed to the 1997-1998 model predictions which were very high, excluding which the average percent error reduces from 17 percent to 9 percent and the RMSE reduces to 4 from 6 ug/L. This discrepancy is likely due to the fact that during 1998, which was a very wet year, much of the TP entering the lake from its tributaries was in particulate form. Following discharge to each lake segment, much of this particulate phosphorus was thus likely lost to benthic sediments, resulting in an attenuation of the response of observed lake TP levels to tributary TP loads. The target error as per the QAPP states that “Tt error targets for this lake modeling exercise are specified as 15% mean error for TP and 5% mean error for chlorides, on a total lake basis.”

**Table 15. Percent Error - Calibration and Validation Statistics (Average and Median over all individual segments) parameterized separately for each segment**

Analyte	Statistic	Calibration (2001-2010)	Validation (1991-2000)
Total Chloride	average error	2.64%	-4.77%
	median error	1.01%	-5.26%
	median R2	0.89	0.82
Total Phosphorus	average error	-1.06%	17.14%
	median error	0.01%	11.84%
	median R2	0.94	0.83
Chlorophyll a	average error	7.05%	3.71%
	median error	4.56%	-0.31%
	median R2	0.85	0.56

**Table 16. Root Mean Square Error (RMSE) - Calibration and Validation Statistics (Average and Median over all individual segments) parameterized separately for each segment**

Analyte	Statistic	Calibration (2001-2010)	Validation (1991-2000)
Total Chloride	average RMSE	0.96	0.96
	median RMSE	1.02	1.00
	median R2	0.89	0.82
Total Phosphorus	average RMSE	2.92	5.70
	median RMSE	2.89	4.07
	median R2	0.94	0.83
Chlorophyll a	average RMSE	0.97	2.42
	median RMSE	0.63	2.16
	median R2	0.85	0.56

Example calibration results for predicted and observed results for the overall lake wide average results are given in Figure 4. BATHTUB results for each of these modeling periods and lake segments are given in the plots in Appendix C. Error bars shown for both the lake observations (red) and the BATHTUB model predictions (black) are in terms of one standard error above and below the observed or predicted mean values. Corresponding TCL and TP error statistics (% error and r-squared) for each 2-year period (1991 through 2010) and for the 10-year calibration period (2001 through 2010) are given in Table 17 and Table 18, respectively. Similarly RMSE statistics for TCL and TP can be found in Table 19 and Table 20.



Under current conditions used for the BATHTUB calibration (2001-2010), lake-wide averaged model error values given in Table 17 and Table 18 for TCL and TP were found to be 1.9 and -1.9 percent, respectively. Corresponding calibration R-squared values were found to be 0.90 and 0.99. These global (lake wide average) error statistics are well within the QA/QC Work Plan error limits established for the current work for the calibration period i.e. current conditions period.

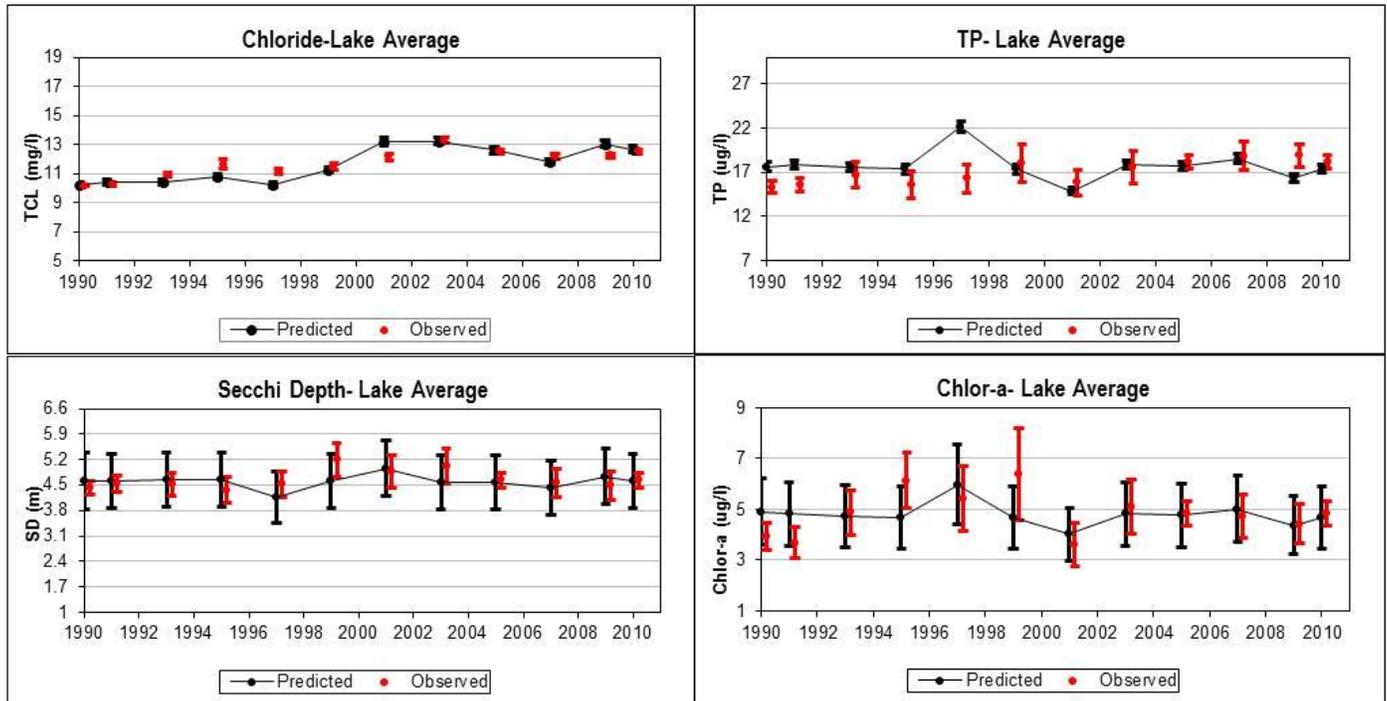


Figure 4. BATHTUB Model Calibration Results – Lake Wide Average.

Table 17. BATHTUB Modeling TCL % Error and R-Squared Statistics

Seq	01-10	91-92	93-94	95-96	97-98	99-00	01-02	03-04	05-06	07-08	09-10
1	-10.3%	8.8%	-7.6%	-10.9%	-18.6%	-13.4%	-0.2%	-10.8%	-11.0%	-17.1%	-6.5%
2	0.0%	-7.6%	-16.0%	-14.1%	-6.0%	4.1%	15.4%	-3.8%	-1.5%	-7.2%	6.1%
3	-0.1%	-0.1%	-4.2%	-6.2%	-6.7%	-0.2%	9.3%	0.0%	-0.5%	-5.0%	3.2%
4	-0.1%	6.2%	-2.4%	-2.4%	-11.0%		9.1%	-0.4%	-0.5%	-4.3%	4.1%
5	-0.2%	5.0%	-2.5%	-6.7%	-8.6%	-1.4%	8.9%	1.2%	-0.5%	-5.4%	3.9%
6	-0.2%	5.9%	-3.0%	-5.5%	-9.6%		8.8%	0.3%	-0.4%	-4.7%	3.9%
7	-0.2%	1.8%	-3.6%	-4.6%	-8.6%	-1.0%	8.7%	0.7%	-0.5%	-4.9%	3.5%
8	-0.1%	3.6%	-2.1%	-2.7%	-6.2%	0.1%	10.3%	0.9%	-0.5%	-5.7%	3.7%
9	11.8%	-5.2%	-14.5%	-10.9%	-5.3%	-5.6%	11.3%	6.2%	11.7%	10.2%	25.6%
10	0.9%	-2.5%	-7.0%	-10.0%	-13.4%	-6.0%	1.8%	-5.0%	1.0%	1.0%	11.5%
11	0.8%	0.0%	-9.5%	-12.8%	-12.0%	-6.5%	0.6%	-5.1%	1.4%	3.8%	6.7%
12	17.3%	1.5%	1.0%	-2.0%	-2.1%	-5.3%	8.9%	1.7%	17.2%	16.0%	24.4%
13	-0.1%	3.3%	-2.2%	-5.7%	-8.0%	-1.6%	12.8%	0.3%	-0.3%	-5.6%	3.4%
Lake	1.2%	1.5%	-4.6%	-7.3%	-8.8%	-2.1%	8.6%	-0.3%	0.9%	-3.1%	6.6%
R <sup>2</sup>	0.94	0.92	0.79	0.78	0.71	0.91	0.78	0.96	0.94	0.91	0.86



Table 18. TP % Error and R-Squared Statistics

Seq	01-10	91-92	93-94	95-96	97-98	99-00	01-02	03-04	05-06	07-08	09-10
1	-0.7%	-21.1%	-2.9%	15.3%	57.7%	7.7%	-24.6%	-3.0%	0.7%	8.7%	6.6%
2	4.7%	12.9%	23.8%	12.5%	60.0%	2.4%	-12.0%	-6.8%	12.2%	20.4%	6.2%
3	-1.5%	15.0%	9.5%	32.6%	38.9%	8.7%	-5.1%	3.4%	1.6%	0.9%	-9.8%
4	-2.7%	11.8%	27.6%	-2.0%	102.8%		-7.0%	12.4%	0.0%	-6.1%	-17.4%
5	-1.8%	2.2%	-4.0%	11.7%	30.1%	10.5%	0.0%	14.8%	-0.2%	1.6%	-14.2%
6	3.8%	4.6%	-0.5%	34.0%	99.6%		0.3%	18.2%	6.2%	5.2%	-7.7%
7	-3.0%	5.8%	-0.8%	28.9%	17.6%	8.8%	-9.9%	14.5%	-1.5%	0.2%	-12.0%
8	16.7%	14.2%	14.2%	35.0%	41.2%	23.8%	10.4%	26.8%	19.5%	22.9%	5.6%
9	-0.4%	10.0%	1.7%	1.3%	42.0%	10.5%	-1.5%	8.8%	0.6%	0.2%	-16.4%
10	-12.0%	12.2%	7.2%	-0.3%	35.5%	-17.8%	-13.9%	-13.4%	-10.2%	-10.9%	-16.4%
11	-0.1%	21.8%	2.1%	5.5%	51.2%	-12.6%	-26.7%	-14.5%	2.2%	21.7%	5.8%
12	0.1%	51.0%	25.5%	22.9%	35.5%	-4.0%	8.6%	9.9%	0.3%	-3.2%	-13.3%
13	-13.9%	-1.0%	-18.2%	2.6%	19.4%	-14.3%	-20.1%	-6.6%	-12.1%	-6.2%	-23.2%
Lake	-4.4%	14.6%	5.5%	11.1%	35.9%	-3.6%	-6.1%	1.5%	-2.4%	-1.6%	-13.4%
R <sup>2</sup>	0.98	0.90	0.94	0.90	0.48	0.95	0.92	0.95	0.98	0.95	0.91



Table 19. TCL Root Mean Square Error Statistics

Seg	01-10	91-92	93-94	95-96	97-98	99-00	01-02	03-04	05-06	07-08	09-10
1	1.85	1.10	1.05	1.71	3.10	2.26	0.03	2.02	1.97	3.12	1.14
2	0.00	1.25	2.70	2.51	0.92	0.63	2.50	0.71	0.27	1.23	1.01
3	0.02	0.01	0.49	0.79	0.81	0.02	1.26	0.01	0.07	0.72	0.46
4	0.02	0.67	0.28	0.30	1.40	0.00	1.22	0.07	0.07	0.61	0.58
5	0.02	0.52	0.28	0.84	1.04	0.18	1.18	0.18	0.07	0.76	0.54
6	0.02	0.63	0.35	0.70	1.21	0.00	1.19	0.05	0.06	0.67	0.56
7	0.02	0.19	0.42	0.57	1.05	0.12	1.16	0.10	0.07	0.69	0.50
8	0.02	0.37	0.23	0.32	0.71	0.01	1.32	0.13	0.06	0.78	0.51
9	1.13	0.49	1.54	1.11	0.51	0.56	1.15	0.66	1.12	0.89	2.25
10	0.09	0.23	0.68	1.00	1.30	0.59	0.19	0.55	0.10	0.09	1.05
11	0.09	0.00	1.04	1.44	1.28	0.70	0.07	0.61	0.15	0.39	0.68
12	1.21	0.12	0.07	0.16	0.16	0.45	0.72	0.14	1.20	1.05	1.66
13	0.01	0.34	0.24	0.68	0.92	0.19	1.57	0.04	0.04	0.75	0.46
R2	0.94	0.92	0.79	0.78	0.71	0.91	0.78	0.96	0.94	0.91	0.86
Overall RMSE	0.69	0.59	1.00	1.12	1.29	0.79	1.23	0.67	0.72	1.14	1.02

Table 20. TP Root Mean Square Error Statistics

Seg	01-10	91-92	93-94	95-96	97-98	99-00	01-02	03-04	05-06	07-08	09-10
1	0.37	12.22	1.62	7.23	23.93	3.78	12.01	1.68	0.37	4.02	3.33
2	1.76	4.20	8.13	4.50	17.40	0.94	4.01	3.07	4.58	6.51	2.42
3	0.25	2.13	1.41	3.92	5.41	1.31	0.68	0.55	0.26	0.16	1.75
4	0.46	1.73	3.48	0.32	9.76	0.00	0.95	1.83	0.00	1.17	3.35
5	0.22	0.25	0.50	1.27	3.18	1.16	0.00	1.56	0.03	0.21	1.91
6	0.51	0.66	0.07	3.63	7.97	0.00	0.04	2.18	0.82	0.73	1.07
7	0.40	0.77	0.11	2.85	2.15	1.04	1.18	1.62	0.20	0.02	1.67
8	2.18	1.93	2.01	3.85	4.90	2.92	1.21	3.32	2.55	3.07	0.79
9	0.05	0.92	0.15	0.12	4.39	1.10	0.16	0.95	0.07	0.02	1.94
10	2.15	1.76	1.04	0.05	5.46	3.26	2.25	2.50	1.83	2.06	2.88
11	0.04	4.94	0.54	1.33	12.85	3.62	8.54	4.52	0.66	6.29	1.53
12	0.03	19.15	10.78	9.37	19.07	1.99	3.53	4.64	0.16	1.62	6.48
13	2.03	0.13	2.78	0.31	2.43	2.16	2.71	0.92	1.77	0.90	3.62
R2	0.98	0.90	0.94	0.90	0.48	0.95	0.92	0.95	0.98	0.95	0.91
Overall RMSE	1.16	6.65	4.04	4.07	11.39	2.36	4.49	2.58	1.65	3.00	2.89



Table 21 presents lake segment average observed and predicted TP and TCL concentrations and CV values for the base calibration period (2001-2010).

**Table 21. Observed and Final Predicted Lake Segment Average TP and TCL concentrations and CV values for base calibration period (2001-2010)**

Lake Segment	TCL				TP			
	Observed		Predicted		Observed		Predicted	
	(mg/L)	CV	(mg/L)	CV	(ug/L)	CV	(ug/L)	CV
1-South Lake B	17.9	0.02	16.1	0.026	50.8	0.04	50.5	0.08
2-South Lake A	17.1	0.015	17.1	0.055	37.5	0.033	39.3	0.08
3-Port Henry	14.4	0.005	14.4	0.029	16.6	0.027	16.4	0.09
4-Otter Creek	14.3	0.006	14.3	0.028	16.7	0.05	16.2	0.09
5-Main Lake	14.2	0.011	14.1	0.024	12.3	0.057	12.1	0.04
6-Shelburne Bay	14.5	0.004	14.4	0.034	13.2	0.021	13.6	0.06
7-Burlington Bay	14.3	0.004	14.3	0.028	13.1	0.024	12.7	0.03
8-Cumberland Bay	13.8	0.004	13.8	0.017	13.1	0.018	15.3	0.11
9-Mallett's Bay	9.5	0.011	10.7	0.038	11.3	0.024	11.2	0.06
10-Northeast Arm	9.9	0.014	9.9	0.069	17.9	0.045	15.8	0.03
11-St Albans Bay	10.8	0.008	10.8	0.033	29.4	0.033	29.4	0.27
12-Missisquoi Bay	7	0.019	8.2	0.031	48.5	0.032	48.5	0.06
13-Isle La Motte	13.5	0.008	13.5	0.019	14.6	0.035	12.5	0.03

Plots of the sensitivity of BATHTUB lake TP predictions to differences in tributary TP load predictions, between the FLUX32 predictions of the current study and those published by VTDEC (Smeltzer 2009), are given in Appendix D. These plots also contain observed TP levels and error bars (plus and minus one standard error), for each lake segment and 2-year simulation time period. Results suggest that differences between these tributary TP load estimation methods yield relatively small differences in BATHTUB model predicted TP levels, within individual lake segments.

## Summary and Conclusions

Several data sets were developed to aid in the development of the Lake Champlain BATHTUB model. These datasets were subsequently used for simulation of total chlorides (TCL), total phosphorous (TP), chlorophyll-*a*, secchi depth and hydraulics, within an interconnected network of 13 Lake Champlain segments, over various historical time-averaging periods. These BATHTUB models were then calibrated, using 20+ years of tributary and lake monitoring data collected between 1990 and 2011 and tributary TP loads published by VTDEC (Smeltzer 2009), for 2-year duration periods between 1991 and 2010.

Daily tributary discharge volumes and TP and CL loads were determined during the current study, using the FLUX program. These FLUX results and tributary TP FLUX results published by VTDEC were input directly to the above spreadsheet method for BATHTUB input dataset generation. Tributary flows and loads determined during the FLUX analysis of the current study were compared to those published by VTDEC, and although generally similar, significant differences were also found. However, BATHTUB predictions using these alternative tributary inputs were found to be similar.

A comprehensive mass balance was first developed for the 2001 to 2010 period data for TCL and TP and was used to aid in the calibration of a 2001 to 2010 average BATHTUB model. The calibrated exchange rates and TP sedimentation factors derived from the mass balance model were then transferred to eleven 2-year period BATHTUB models starting in 1991, 1993, 1995, 1997, 1999, 2001, 2003, 2005, 2007, 2009 and 2010. The six 2-year models during the period from 2001 to 2010 were considered as the calibration period and the five 2-year models from 1991 to 1999 were used to guide the validation of the model results.

Model results for total chloride and chlorophyll a indicate that the percent error magnitudes remain small during the calibration and validation period; however, for total phosphorus there is an increase in the total phosphorus error during the validation period. In addition very wet-years such as those observed in 1997-1998 also result in these over predictions.

The calibrated BATHTUB model will be used to determine allocations and phosphorus reductions under EPA's revised Lake Champlain Total Phosphorus TMDL. Table 22 presents the total annual phosphorus loading to Lake Champlain by segment and state/province.

**Table 22. Bathtub Predicted Annual Phosphorus Loading by Lake Segment and State/Province (2001-2010 Base Period)**

Lake Segment	TP (metric tons / yr)			
	Vermont	Quebec	New York	Total
1	51.1	0.0	39.4	90.5
2	26.4	0.0	24.4	50.7
3	7.0	0.0	8.4	15.4
4	140.5	0.0	0.4	140.9
5	162.0	0.0	64.9	226.8
6	10.2	0.0	0.0	10.2
7	4.5	0.0	0.0	4.5
8	0.0	0.0	42.0	42.0
9	56.4	0.0	0.0	56.4
10	17.7	0.5	0.0	18.2
11	13.9	0.0	0.0	13.9
12	136.2	72.2	0.0	208.4
13	4.1	4.6	34.2	42.9
<b>Grand Total</b>	<b>630.0</b>	<b>77.3</b>	<b>213.6</b>	<b>920.9</b>

Note: Bathtub predicted load totals were distributed between states based on the relative TP load contribution from the SWAT model for areas in VT, NY and QC



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