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An Applied General Equilibrium Model for the Analysis of Environmental Policy: SAGE v1.0 Technical Documentation

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An Applied General Equilibrium Model for the Analysis of Environmental Policy: SAGE v1.0 Technical Documentation*

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Abstract

SAGE is an applied general equilibrium model of the United States economy developed for the analysis of environmental regulations and policies. It is an intertemporal model with perfect foresight, resolved at the sub-national level. Each of the nine regions in the model, representing the nine census divisions, has five households based on income quintiles. A single government agent levies taxes on labor earnings, capital earnings, production, and consumption. As with many applied general equilibrium models used for the analysis of U.S. environmental and energy policies, the baseline is calibrated to the Energy Information Administration's Annual Energy Outlook. The model is solved as mixed complementarity problem (MCP) using the General Algebraic Modeling System (GAMS).

*The views expressed in this paper are those of the authors and do not necessarily represent those of the U.S. EPA. No Agency endorsement should be inferred.

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1 Introduction

SAGE is an applied general equilibrium model of the United States economy developed for the analysis of environmental regulations and policies.[1] It is an intertemporal model with perfect foresight, resolved at the sub-national level. Each of the nine regions in the model, representing the nine census divisions, has five households based on income quintiles. A single government agent levies taxes on labor earnings, capital earnings, production, and consumption. As with many applied general equilibrium models used for the analysis of U.S. environmental and energy policies, the baseline is calibrated to the Energy Information Administration's Annual Energy Outlook. The model is solved as mixed complementarity problem (MCP) using the General Algebraic Modeling System (GAMS).

In the following section, technical details on the structure of the model are presented. Section 3 describes the model's calibration. Section 4 discusses the solution algorithm. A more general description of the model and sensitivity analyses are provided in Marten et al. (2018).

2 Model Structure

SAGE solves for the set of relative prices that return the economy to equilibrium after the imposition of a policy or other shock, such that all markets clear. This section describes the model's basic structure by first defining the markets in the model, followed by how firms, households, and the government are represented. The section concludes by describing the market clearance conditions that are used to determine equilibrium, where supply equals demand in all markets.

2.1 Trade

The United States is represented as a small open economy, with perfectly elastic demand for its international exports and perfectly elastic supply for international imports. Intra-national trade is pooled at the national level. That is there exists a single market clearing price for commodities traded across regions, independent of the region of origin or destination. This structure for intra-national trade is similar to other CGE models with subnational detail, such as Balistreri and Rutherford (2001) and Rausch et al. (2011). There are nine subnational regions in the model matrching the nine U.S. Census divisions (see Figure 1). Primary factors are not explicitly mobile across regions.

Within a region, goods from different origins are aggregated using the Armington specification (Armington, 1969). The Armington aggregate is based on first bundling regional output with intra-national imports and then combining that bundle with international imports. A constant elasticity of transformation (CET) function is used to differentiate regional output between different destination markets. This structure is presented in Figure 2.

More specifically, the Armington aggregate is defined as

$$a_{t,r,s} = a0_{r,s}\Bigg\{ cs\_nf_{r,s}\left(\frac{m_{t,r,s,ftrd}}{m0_{t,r,s,ftrd}}\right)^{\frac{se\_nf-1}{se\_nf}} + (1-cs\_nf_{r,s})\Bigg[cs\_dn_{r,s}\left(\frac{m_{t,r,s,dtrd}}{m0_{t,r,s,dtrd}}\right)^{\frac{se\_dn-1}{se\_dn}} + (1-cs\_dn_{r,s})\left(\frac{d_{t,r,s}}{d0_{t,r,s}}\right)^{\frac{se\_dn-1}{se\_dn}}\Bigg]^{\frac{(se\_nf-1)se\_dn}{se\_nf(se\_dn-1)}}\Bigg\}^{\frac{se\_nf}{se\_nf-1}}, \quad (1)$$

[1] We use a recursive naming convention, where SAGE stands for SAGE is an Applied General Equilibrium model.

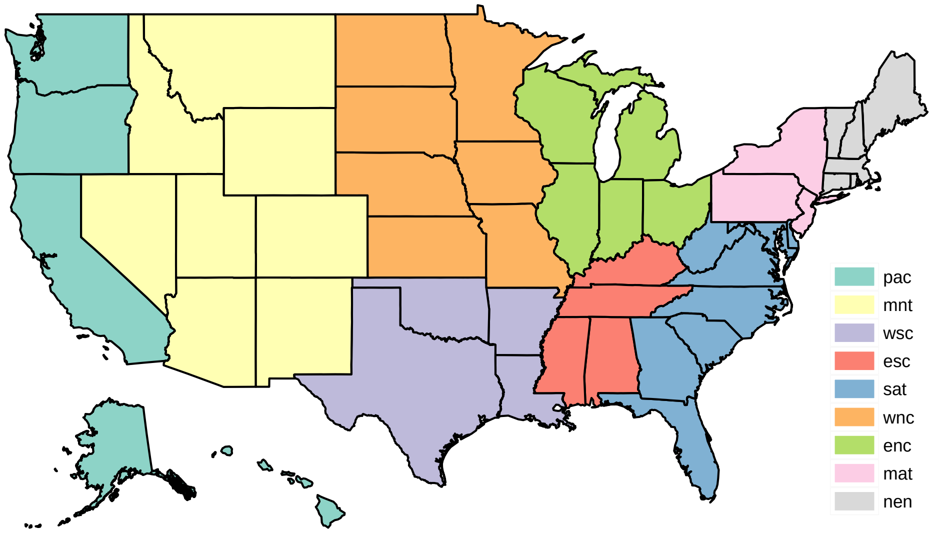

Figure 1: SAGE Regions

where $a_{t,r,s}$ is the Armington composite in period t and region r for commodity s, $m_{t,r,s,trd}$ is imports from market trd, $d_{t,r,s}$ is domestic production consumed locally/.[2] The national market is denoted $dtrd$ and the international market is denoted $ftrd$. The parameter $cs\_nf_{r,s}$ represents the international imports share of the Armington composite and $cs\_dn_{r,s}$ represents the share of national imports in the domestic-national composite. The substitution elasticity between international imports and the domestic-national composite is se_nf and the substitution elasticity between domestic production and national imports is se_dn. The inputs into the Armington aggregate are determined based on minimizing the price of the composite good, $pa_{t,r,s}$, given the price in the domestic market, $pd_{t,r,s}$, the price in the national market, $pn_{t,s}$, and the price of foreign exchange, pfx_t.

The CET function to differentiate domestic output across destination markets is defined as

$$\begin{aligned} y_{t,r,s} + y\_ex_{t,r,s} = (y0_{t,r,s} + y\_ex0_{t,r,s}) \Bigg[& cs\_dx_{r,s,d} \left(\frac{d_{t,r,s}}{d0_{t,r,s}} \right)^{\frac{te\_dx-1}{te\_dx}} \\ & + cs\_dx_{r,s,dtrd} \left(\frac{x_{t,r,s,dtrd}}{x0_{t,r,s,dtrd}} \right)^{\frac{te\_dx-1}{te\_dx}} , \\ & + cs\_dx_{r,s,ftrd} \left(\frac{x_{t,r,s,ftrd}}{x0_{t,r,s,ftrd}} \right)^{\frac{te\_dx-1}{te\_dx}} \Bigg]^{\frac{te\_dx}{te\_dx-1}} , \end{aligned} \tag{2}$$

where $y_{t,r,s}$ is output from production with new capital, $y\_ex_{t,r,s}$ is output from production with extant capital, $x_{t,r,s,trd}$ is exports to market trd, $cs\_dx_{r,s,mkt}$ is the share of output destined for market mkt, and te_dx is the transformation elasticity. Within the production possibilities frontier represented by

[2]Throughout this document a 0 trailing a variable name denotes the value in the benchmark year, benchmark cost shares have the prefix cs, and substitution elasticities have the prefix se. In the model, most substitution elasticities vary across sectors as discussed in further detail in Section 3. However, to simply the exposition, in this document we forgo the sector subscript on substitution elasticities.

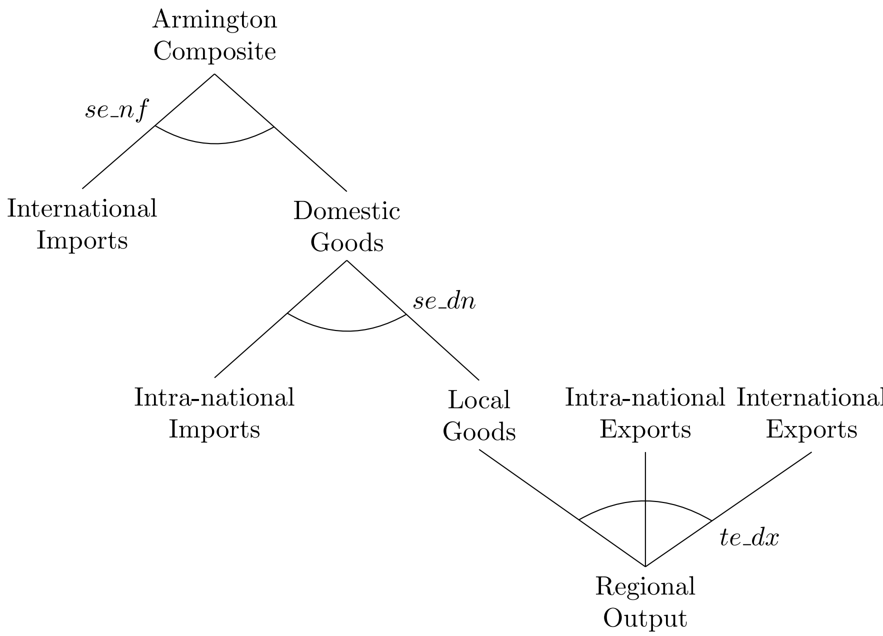

Figure 2: Armington Trade Specification

equation (2), firms select the shares of production destined for each market based on maximizing the price of output, $py_{t,r,s}$, given the price of the commodity in the different destination markets.

2.2 Production

Production in the model is aggregated to 23 sectors, with greater detail in the manufacturing and energy. The sectors in the model and associated NAICS codes are presented in Table 1.

2.2.1 Manufacturing and Service Sectors

In SAGE, perfectly competitive firms minimize costs of production subject to market prices and a given production technology. Nested constant elasticity of substitution (CES) production functions have become widely used in applied general equilibrium modeling (Brockway et al., 2017), and this is particularly true in the case of CGE models used to analyze energy and environmental policies. Similarly, SAGE makes use of nested CES functions (in calibrated share form) to define the production functions for the sectors represented. The policy response of CGE models based on nested CES production functions may be sensitive to the ordering of the nests, as this choice defines separability of the production functions amongst inputs (Lecca et al., 2011). Therefore, there has been much discussion about the hierarchy for the nested CES production functions, particularly with regards to the capital, K, labor, L, and energy, E, inputs. Much of this discussion has been based on heuristics, although the empirical work of Van der Werf (2008) is a notable exception. Van der Werf (2008) studied the fit of different nesting structures given historical production data for 12 OECD countries between 1978 and 1996. Van der Werf (2008) finds that the nesting structure combining K and L in the lower nest and the KL bundle with E in the top nest, denoted $KL(E)$, provides a significantly better fit to the data compared to the other possible nesting structures. Furthermore, Van der Werf (2008) finds that the structure combining K and E in

Table 1: Model Sectors

Abbreviation	Description	NAICS Codes	NEMS IDM Code
agf	Agriculture, forestry, fishing and hunting	11	1, 2
gas	Natural gas extraction and distribution	211*, 213111*, 213112*, 2212,	4*
cru	Crude oil extraction	211*, 213111*, 213112*	4*
col	Coal mining	2121, 213113	3
min	Metal ore and nonmetallic mineral mining	2122, 2123, 213114, 213115	5
ele	Electric power	2211	NA
wsu	Water, sewage, and other utilities	2213	NA
con	Construction	23	6
fbm	Food and beverage manufacturing	311, 312	7
wpm	Wood and paper product manufacturing	321, 322	8, 19
ref	Petroleum refineries	32411	NA
chm	Chemical manufacturing	325	9
prm	Plastics and rubber products manufacturing	326	20
cem	Cement	32731	22
pmm	Primary metal manufacturing	331	12, 13
fmm	Fabricated metal product manufacturing	332	14
cpu	Electronics and technology	334, 335	16, 18
tem	Transportation equipment manufacturing	336	17
bom	Balance of manufacturing	3122, 313, 314, 316, 323, 32412, 3271, 3272, 32732, 32733, 32739, 3274, 3279, 333, 337, 339	10, 15, 21, 23
trn	Non-Truck Transportation	481, 482, 483, 485, 486, 4869, 487, 488, 491, 492, 493	NA
ttn	Truck transportation	484	NA
srv	Services	42, 44, 45, 51, 52, 53, 54, 55, 56, 61, 624, 71, 72, 81	NA
hlt	Healthcare services	621, 622	NA

* Crude oil and natural gas extraction is included as a single sector in the benchmark data, however, we disaggregate this activity into separate sectors for crude oil and natural gas. Details are available in Section 3.1.1.

the lower nest provided the worst fit for the data, a finding that has been corroborated in other single country contexts (e.g., Dissou et al. (2015); Ha et al. (2012); Kemfert (1998)). Other multi- and single-country studies have found that the $KE(L)$ nesting structure may fit the data as well as the $KL(E)$ structure at the aggregate national level (e.g., Markandya and Pedroso-Galinato (2007); Su et al. (2012)). However, Kemfert (1998) finds that in cases where the $KE(L)$ nesting structure finds support at the aggregate national level the specification may actually provide a worse fit than the $KL(E)$ structure when disaggregated sectoral production functions are estimated. Therefore, we use a structure that combines primary factors K and L in a lower nest, where that value-added bundle is then combined with an energy composite. At the top level of the production function the $KL(E)$ composite is combined with material inputs. This structure is similar to other CGE models used to analyze energy and environmental policies (e.g., Paltsev et al. (2005); Rausch et al. (2011); Capros et al. (2013); Cai et al. (2015)).

For the energy composite we also use a nested CES function to represent available production technologies. Initial work using energy-explicit CGE models typically combined all energy sources, including primary energy sources and electricity, in a single nest, commonly with a unit substitution elasticity (e.g., Borges and Goulder (1984)). Subsequent efforts separated electricity from other primary energy sources in a two-nest CES structure that defined the energy composite (e.g., Babiker et al. (1997); Paltsev et al. (2005); Rausch et al. (2011) Böhringer et al. (2018)). Some recent models have even gone a step further using a three-level CES nest to further disaggregate the primary energy composite to assume separability between some of the fossil-fuel use decisions in the cost-minimization problem (e.g., Burniaux and Truong (2002); Chateau et al. (2014); Ross (2014)). However, within this class of models the three-level CES nesting structure is not consistent between models and evidence of weak seperability in data is lacking empirically (Serletis et al., 2010a). Therefore, SAGE applies the two level energy nesting with the bottom level nest combining refined petroleum products (or by-products), coal, and natural gas. The second level nest combines the primary energy composite with electricity. This nesting structure is presented in Figure 3.

More specifically, the production function for manufacturing goods and services produced with new capital is

$$y_{t,r,s} = y0_{r,s}\left[cs\_klem_{r,s}\left(\frac{mat_{t,r,s}}{mat0_{t,r,s}}\right)^{\frac{se\_klem-1}{se\_klem}} + (1-cs\_klem_{r,s})\left(\frac{kle_{t,r,s}}{kle0_{t,r,s}}\right)^{\frac{se\_klem-1}{se\_klem}}\right]^{\frac{se\_klem}{se\_klem-1}}, \quad (3)$$

where $mat_{t,r,s}$ is the materials bundle

$$mat_{t,r,s} = mat0_{t,r,s}\min\left(\frac{id_{t,r,agf,s}}{id0_{t,r,agf,s}},\ldots,\frac{id_{t,r,srv,s}}{id0_{t,r,srv,s}}\right), \quad (4)$$

$id_{t,r,ss,s}$ is the demand for intermediate good ss, $kle_{t,r,s}$ is the energy and value added composite

$$kle_{t,r,s} = kle0_{r,s}\left[cs\_kle_{r,s}\left(\frac{ene_{t,r,s}}{ene0_{t,r,s}}\right)^{\frac{se\_kle-1}{se\_kle}} + (1-cs\_kle_{r,s})\left(\frac{kl_{t,r,s}}{kl0_{t,r,s}}\right)^{\frac{se\_kle-1}{se\_kle}}\right]^{\frac{se\_kle}{se\_kle-1}}, \quad (5)$$

$ene_{t,r,s}$ is the electricity and primary energy composite

$$ene_{t,r,s} = ene0_{r,s}\left[cs\_ene_{r,s}\left(\frac{en_{t,r,s}}{en0_{t,r,s}}\right)^{\frac{se\_ene-1}{se\_ene}} + (1-cs\_ene_{r,s})\left(\frac{id_{t,r,ele,s}}{id0_{t,r,ele,s}}\right)^{\frac{se\_ene-1}{se\_ene}}\right]^{\frac{se\_ene}{se\_ene-1}}, \quad (6)$$

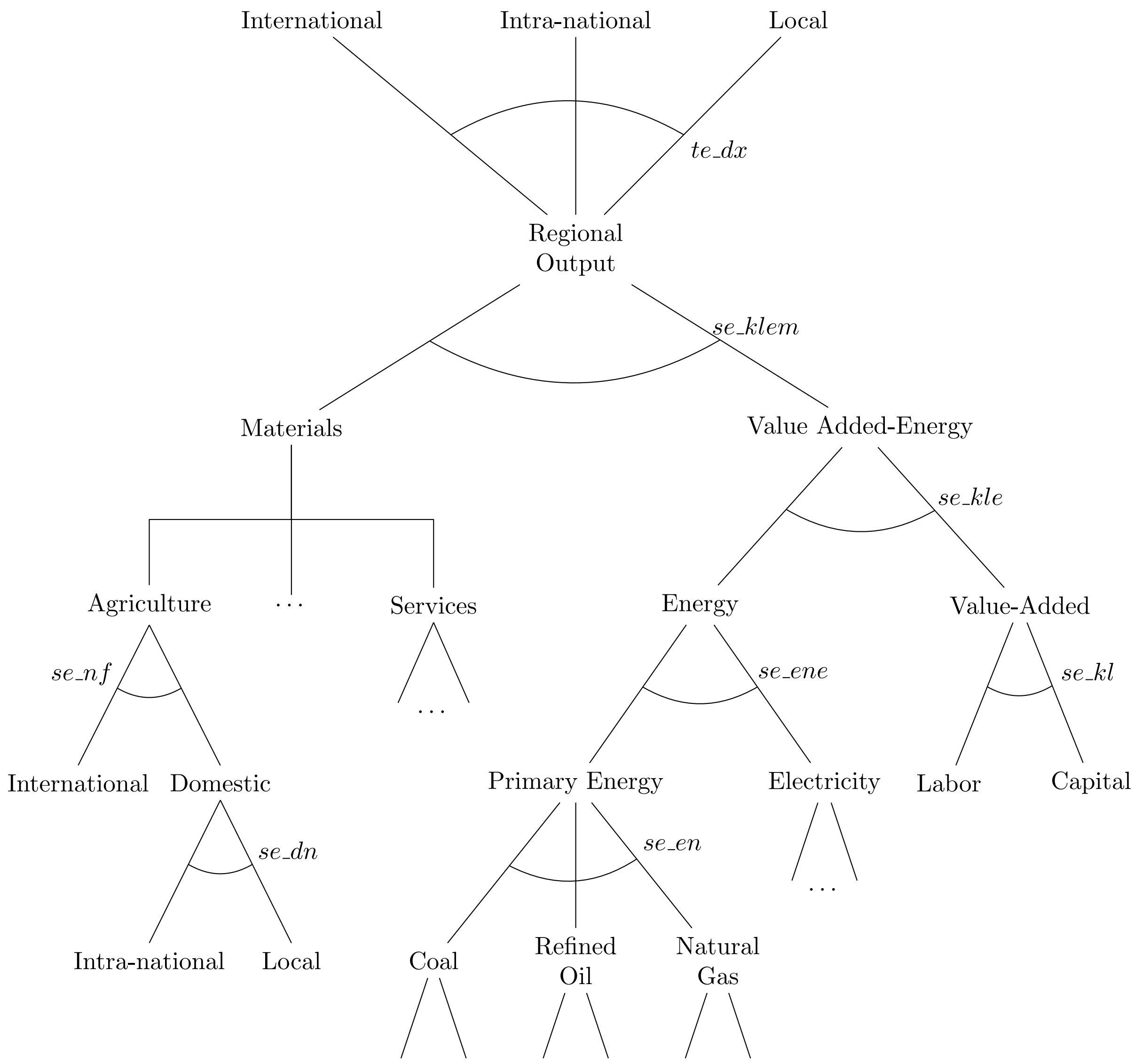

Figure 3: Manufacturing and Services Production Functions

$en_{t,r,s}$ is the primary energy composite

$$en_{t,r,s} = en0_{r,s}\left[cs\_en_{r,col,s}\left(\frac{id_{t,r,col,s}}{id0_{t,r,col,s}}\right)^{\frac{se\_en-1}{se\_en}} + cs\_en_{r,ref,s}\left(\frac{id_{t,r,ref,s}}{id0_{t,r,ref,s}}\right)^{\frac{se\_en-1}{se\_en}} + cs\_en_{r,gas,s}\left(\frac{id_{t,r,gas,s}}{id0_{t,r,gas,s}}\right)^{\frac{se\_en-1}{se\_en}}\right]^{\frac{se\_en}{se\_en-1}}, \quad (7)$$

$kl_{t,r,s}$ is the value added composite

$$kl_{t,r,s} = kl0_{r,s}\left[cs\_kl_{r,s}\left(\frac{kd_{t,r,s}}{kd0_{t,r,s}}\right)^{\frac{se\_kl-1}{se\_kl}} + (1 - cs\_kl_{r,s})\left(\frac{ld_{t,r,s}}{ld0_{t,r,s}}\right)^{\frac{se\_kl-1}{se\_kl}}\right]^{\frac{se\_kl}{se\_kl-1}}, \quad (8)$$

$kd_{t,r,s}$ is demand for new capital, $ld_{t,r,s}$ is demand for labor, parameters with the prefix cs are the relevant cost shares in the benchmark year, and parameters with the prefix se are the relevant substitution elasticities.

Markets are assumed to be perfectly competitive, such that firms are price takers. Given market prices, firms seek to maximize profits

$$(1 - ty_{t,r,s})\, py_{t,r,s} y_{t,r,s} - \sum_{ss} pa_{t,r,ss} id_{t,r,ss,s} - (1 + tk_{t,r})\, pr_{t,r} kd_{t,r,s} - (1 + tl_{t,r})\, pl_{t,r} ld_{t,r,s}, \quad (9)$$

where $py_{t,r,s}$ is the output price based on maximizing returns across destination markets per (2), $pa_{t,r,s}$ is the price of the Armington composite, $pr_{t,r}$ is the rental rate for new capital, $pl_{t,r}$ is the wage rate, and $ty_{t,r,s}$, $tk_{t,r}$, and $tl_{t,r}$ are ad valorem taxes on output, capital, and labor, respectively.

2.2.2 Resource Extraction, Agriculture, and Forestry Sectors

The resource extraction sectors (crude oil, natural gas, coal, and mining) have an additional primary factor input, in this case representing the finite natural resource. In many cases, models have typically included this resource in a top-level nest with a bundle of non-resource inputs (e.g., Ross (2005); Paltsev et al. (2005); Sue Wing (2006); Rausch et al. (2011); Capros et al. (2013); Ross (2014); Böhringer et al. (2018)). While some models allow for substitution between materials, energy, and value-added in resource extraction sectors (e.g., Sue Wing et al. (2011); Capros et al. (2013)), other models treat energy, labor, and capital as Leontief inputs (e.g., Ross (2014)), although in most cases there is some substitutability allowed between labor and capital (e.g., Ross (2005); Paltsev et al. (2005); Sue Wing (2006); Rausch et al. (2011)). Recent empirical evidence suggests non-zero and statistically significant substitution elasticities between inputs in resource extraction industries (Young (2013); Koesler and Schymura (2015)). Therefore, we maintain the same structure as in the standard production nesting albeit for the addition of a fixed resource. The structure of the production functions for the fossil fuel extraction sectors is presented in Figure 4.

We model the agriculture and forestry sectors using a similar production function, with land as a fixed factor input. We recognize that there has been an ongoing discussion in the literature related to the degree of flexibility required by a production function to capture the separability, or lack thereof, observed in empirical studies of agricultural sectors (e.g., Higgs and Powell (1990); Zahniser et al. (2012); Simola (2015)). However, the decreasing returns to scale nature of production in the sector, as captured in Figure 4, is common among approaches, independent of the nesting structure applied.

For resource extraction, agriculture, and forestry sectors the specific form of the production function

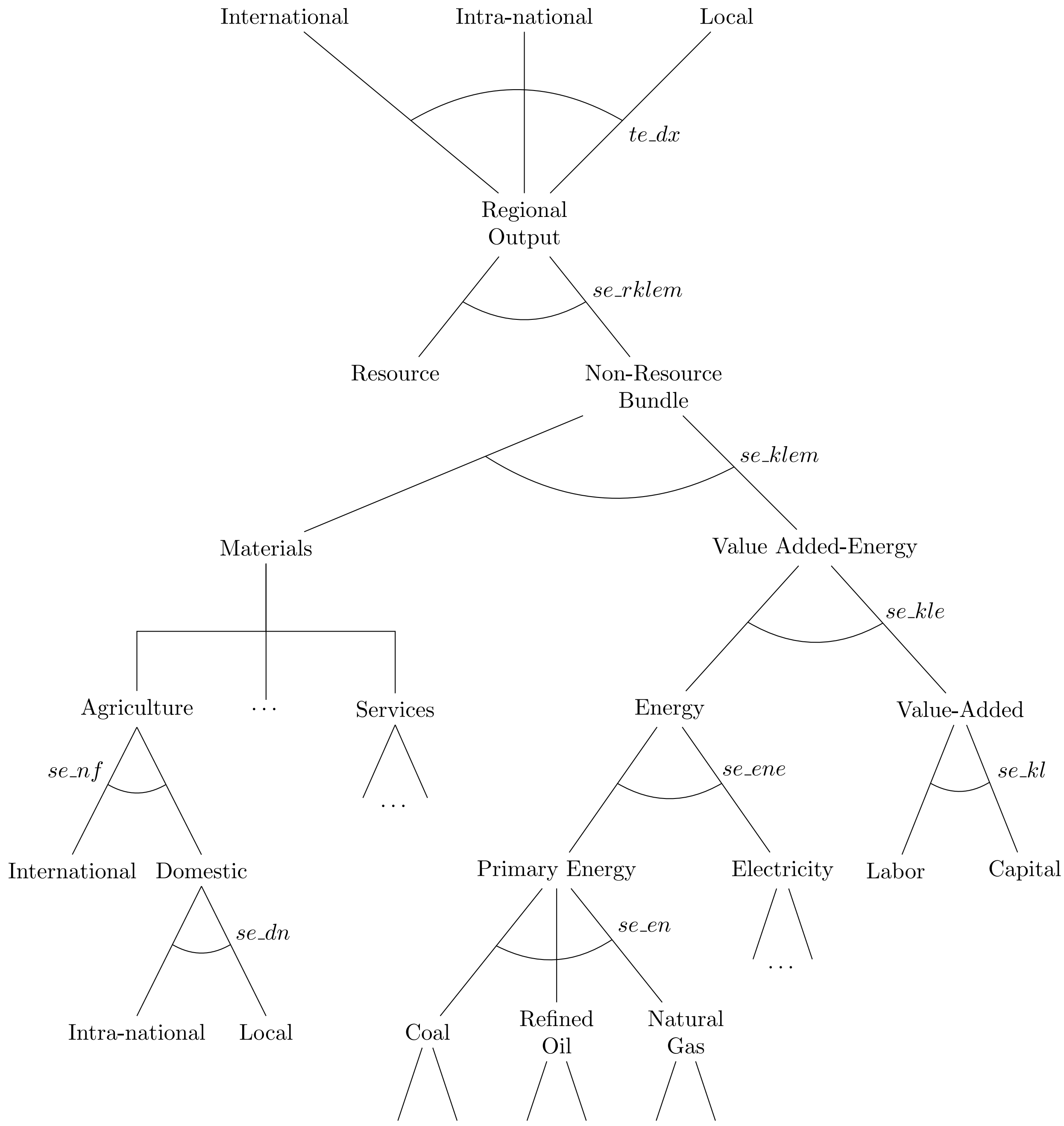

Figure 4: Resource Extraction, Agriculture, and Forestry Production Functions

is

$$y_{t,r,s} = y0_{r,s}\left[cs\_rklem_{r,s}\left(\frac{res_{t,r,s}}{res0_{t,r,s}}\right)^{\frac{se\_rklem-1}{se\_rklem}} + (1 - cs\_rklem_{r,s})\left(\frac{klem_{t,r,s}}{klem0_{t,r,s}}\right)^{\frac{se\_rklem-1}{se\_rklem}}\right]^{\frac{se\_rklem}{se\_rklem-1}}, \tag{10}$$

where

$$klem_{t,r,s} = klem0_{r,s}\left[cs\_klem_{r,s}\left(\frac{mat_{t,r,s}}{mat0_{t,r,s}}\right)^{\frac{se\_klem-1}{se\_klem}} + (1 - cs\_klem_{r,s})\left(\frac{kle_{t,r,s}}{kle0_{t,r,s}}\right)^{\frac{se\_klem-1}{se\_klem}}\right]^{\frac{se\_klem}{se\_klem-1}}, \tag{11}$$

and $mat_{t,r,s}$ and $kle_{t,r,s}$ are defined in (4)-(8). The fixed factors, $res_{t,r,s}$, are sector specific and in the baseline fixed at the benchmark level, $res_{t,r,s} = res0_{r,s}\ \forall t$.

The resource extraction, agriculture, and forestry markets are also assumed to be perfectly competitive, such that firms are price takers. Given market prices, firms seek to maximize profits

$$\begin{aligned}(1 - ty_{t,r,s})\, py_{t,r,s} y_{t,r,s} - \sum_{ss} pa_{t,r,ss} id_{t,r,ss,s} - (1 + tk_{t,r})\, pr_{t,r} kd_{t,r,s} - (1 + tl_{t,r})\, pl_{t,r} ld_{t,r,s}\\ - (1 + tk_{t,r})\, pres_{t,r,s} res_{t,r,s},\end{aligned} \tag{12}$$

where $pres_{t,r,s}$ is the price of the fixed factor resource, which is assumed to face the same ad valorem tax rate as capital.

2.3 Capital Markets

To better represent limitations associated with transitioning existing capital stock between sectors, the model considers two capital vintages: existing stock in the benchmark year and new capital formed after the benchmark year. Production with new capital has the flexibility described in Figure 3. Production with extant capital has a Leontief production structure, as shown in Figure 5.[3] For a profit maximizing firm this means that output of commodity s using extant capital is

$$y\_ex_{t,r,s} = y\_ex0_{r,s}\frac{kd\_ex_{t,r,s}}{kd0\_ex_{r,s}} \tag{13}$$

and demand for intermediate good ss and labor to used with extant capital will be

$$id\_ex_{t,r,ss,s} = id\_ex0_{r,ss,s}\frac{kd\_ex_{t,r,s}}{kd0\_ex_{r,s}} \tag{14}$$

and

$$ld\_ex_{t,r,s} = ld\_ex0_{r,s}\frac{kd\_ex_{t,r,s}}{kd0\_ex_{r,s}}. \tag{15}$$

In our putty-clay specification, extant capital is primarily sector specific, although there is limited potential to shift extant capital across sectors at a cost. This is to capture the observed maintenance of extant capital beyond its expected lifespan when vintage differentiated regulation targets only production associated with new capital. To capture this characteristic, sector specific extant capital, $kd\_ex_{t,r,s}$ is determined by a CET function that transforms a region's extant capital, $k\_ex_{t,r}$, with elasticity te_k_ex. More specifically, given the rental rates for sector specific extant capital the returns to stock of extant

[3] Given the Leontief structure of the production function with extant capital, the nesting pictured in Figure 5 is unnecessary but is retained to make the figure more readable.

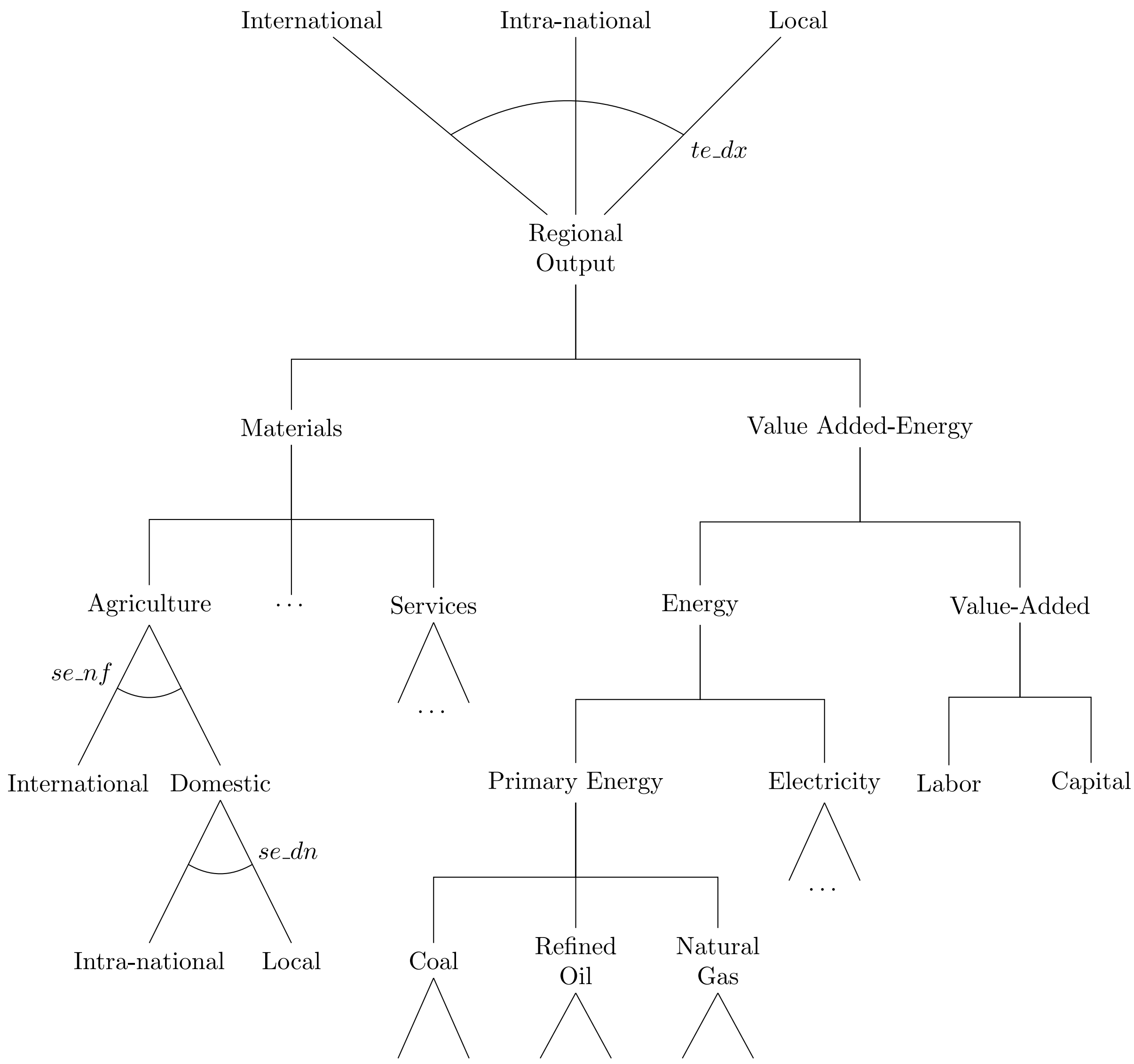

Figure 5: Manufacturing and Services Production Functions with Extant Capital

Table 3: SAGE Representative Households

Household	Benchmark Income
hh1	$\leq$ \$25,000
hh2	\$25,00-\$50,000
hh3	\$50,000-\$75,000
hh4	\$75,000-\$150,000
hh5	$\geq$ \$150,000

capital are maximized subject to the production possibilities frontier

$$k\_ex_{t,r} = k\_ex0_r \left[\sum_s cs\_kd\_ex_{r,s} \left(\frac{kd\_ex_{t,r,s}}{kd\_ex0_{t,r,s}} \right)^{\frac{te\_k\_ex-1}{te\_k\_ex}} \right]^{\frac{te\_k\_ex}{te\_k\_ex-1}}. \tag{16}$$

Sectors associated with a fixed resource, as described in Section 2.2.2, do not have vintaged capital. For those sectors already exhibit decreasing returns to scale, and the own price elasticity of supply is calibrated to empirical estimates as described in Section 3.3.3.

Capital, regardless of vintage, depreciates at rate δ, such that the laws of motion are

$$k_{t+1,r} = (1-\delta)k_{t,r} + inv_{t,r} \tag{17}$$

and

$$k\_ex_{t+1,r} = (1-\delta)k\_ex_{t,r}, \tag{18}$$

where $inv_{t,r}$ is regional investment in year t. Formation of physical capital is assumed to be a Leontief process such that

$$inv_{t,r} = inv0_r \min_s \left(\frac{i_{t,r,s}}{i0_{r,s}} \right), \tag{19}$$

where $i_{t,r,s}$ is investment demand for commodity s.

2.4 Households

Each region has 5 representative households differentiated by benchmark income. The benchmark income for the representative households is presented in Table 3. Based on the underlying economic data in our social accounting matrix, these represent the closest approximation of national income quintiles for the benchmark year.

Each representative household seeks to maximize intertemporal welfare defined for household h in region r as

$$W_{r,h} = \sum_{t=0}^{\infty} \beta^t n_{t,r,h} u \left(\frac{cl_{t,r,h}}{n_{t,r,h}} \right), \tag{20}$$

where β is the discount factor, $n_{t,r,h}$ are the number of households represented by this agent, $cl_{t,r,h}$ is the consumption-leisure composite, and $u(\cdot)$ is the intra-temporal utility function. The discount rate is defined as

$$\beta = \frac{1}{1+\rho}, \tag{21}$$

where ρ is the pure rate of time preference. Households seek to maximize welfare in (20) subject to a budget constraint

$$
\begin{aligned}
pk_{t+1,r}kh_{t+1,r,h} + pcl_{t,r,h}cl_{t,r,h} =& pk_{t,r}kh_{t,r,h} + pl_{t,r}te_{t,r,h} + pr\_ex\_agg_{t,r}kh\_ex_{t,r} \\
&+ \sum_{s} pres_{t,r,s}resh_{t,r,s} + pfx_t bopdef_{t,r,h} + cpi_t tran_{t,r,h}'
\end{aligned}
\tag{22}
$$

where $kh_{t,r,h}$ is the stock of new capital owned by household h, $kh\_ex_{t,r}$ is their stock of extant capital, $resh_{t,r,s,h}$ is their stock of fixed resource used by sector s, $bopdef_{t,r,h}$ is their share of returns on government or foreign debt, $tran_{t,r,h}$ are net government transfers, $pcl_{t,r,h}$ is the cost of full consumption (consumption and leisure), $te_{t,r,h}$ is the household's effective time endowment, $pr\_ex\_agg_{t,r}$ is the value of extant capital stock, cpi_t is the consumer price index

$$
cpi_t = \frac{\sum_{r,s,h} (1 + tc_{t,r})\, pa_{t,r,s} cd_{t,r,s,h}}{\sum_{r,s,h} cd_{t,r,s,h}},
\tag{23}
$$

and $cd_{t,r,s,h}$ is demand for commodity s.

The intra-temporal utility function is isoelastic, such that

$$
u\left(cl_{t,r,h}\right) = \frac{cl_{t,r,h}^{1-\eta}}{1-\eta},
\tag{24}
$$

where η represents the inverse of the intertemporal substitution elasticity of full consumption. Intra-temporal household preferences are defined by a nested CES utility function as presented in Figure 6, following Rausch et al. (2011). The representative households select consumption of energy and non-energy goods, which are then combined with transportation to form the market consumption aggregate. The aggregate consumption bundle is then combined with leisure in the top-level nest of the utility function. More information about the inclusion of leisure and calibration of the substitution elasticity between consumption and leisure is presented in Section 3.3.4.

More specifically, intra-temporal household preferences over full consumption are defined as

$$
cl_{t,r,h} = cl0_{r,h} \left[cs\_cl_{r,h} \left(\frac{c_{t,r,h}}{c0_{t,r,h}} \right)^{\frac{se\_cl-1}{se\_cl}} + (1 - cs\_cl_{r,h}) \left(\frac{leis_{t,r,h}}{leis0_{t,r,h}} \right)^{\frac{se\_cl-1}{se\_cl}} \right]^{\frac{se\_cl}{se\_cl-1}},
\tag{25}
$$

where $leis_{t,r,h}$ is leisure and $c_{t,r,h}$ is the final goods consumption composite

$$
c_{t,r,h} = c0_{r,h} \left[cs\_c_{r,h} \left(\frac{cd_{t,r,trn,h}}{cd0_{t,r,trn,h}} \right)^{\frac{se\_c-1}{se\_c}} + (1 - cs\_c_{r,h}) \left(\frac{cme_{t,r,h}}{cme0_{t,r,h}} \right)^{\frac{se\_c-1}{se\_c}} \right]^{\frac{se\_c}{se\_c-1}},
\tag{26}
$$

where $cme_{t,r,h}$ is the non-transportation goods and energy composite

$$
cme_{t,r,h} = cme0_{r,h} \left[cs\_cme_{r,h} \left(\frac{cm_{t,r,h}}{cm0_{t,r,h}} \right)^{\frac{se\_cme-1}{se\_cme}} + (1 - cs\_cme_{r,s}) \left(\frac{cene_{t,r,h}}{cene0_{t,r,h}} \right)^{\frac{se\_cme-1}{se\_cme}} \right]^{\frac{se\_cme}{se\_cme-1}},
\tag{27}
$$

$cm_{t,r,s}$ is the non-transportation and non-energy composite

$$
cm_{t,r,h} = cm0_{r,h} \left[\sum_{s \in scm} cs\_cm_{r,s,h} \left(\frac{cd_{t,r,s,h}}{cd0_{t,r,s,h}} \right)^{\frac{se\_cm-1}{se\_cm}} \right]^{\frac{se\_cm}{se\_cm-1}},
\tag{28}
$$

scm is the set of non-transportation and non-energy commodities, $cene_{t,r,s}$ is the electricity and primary

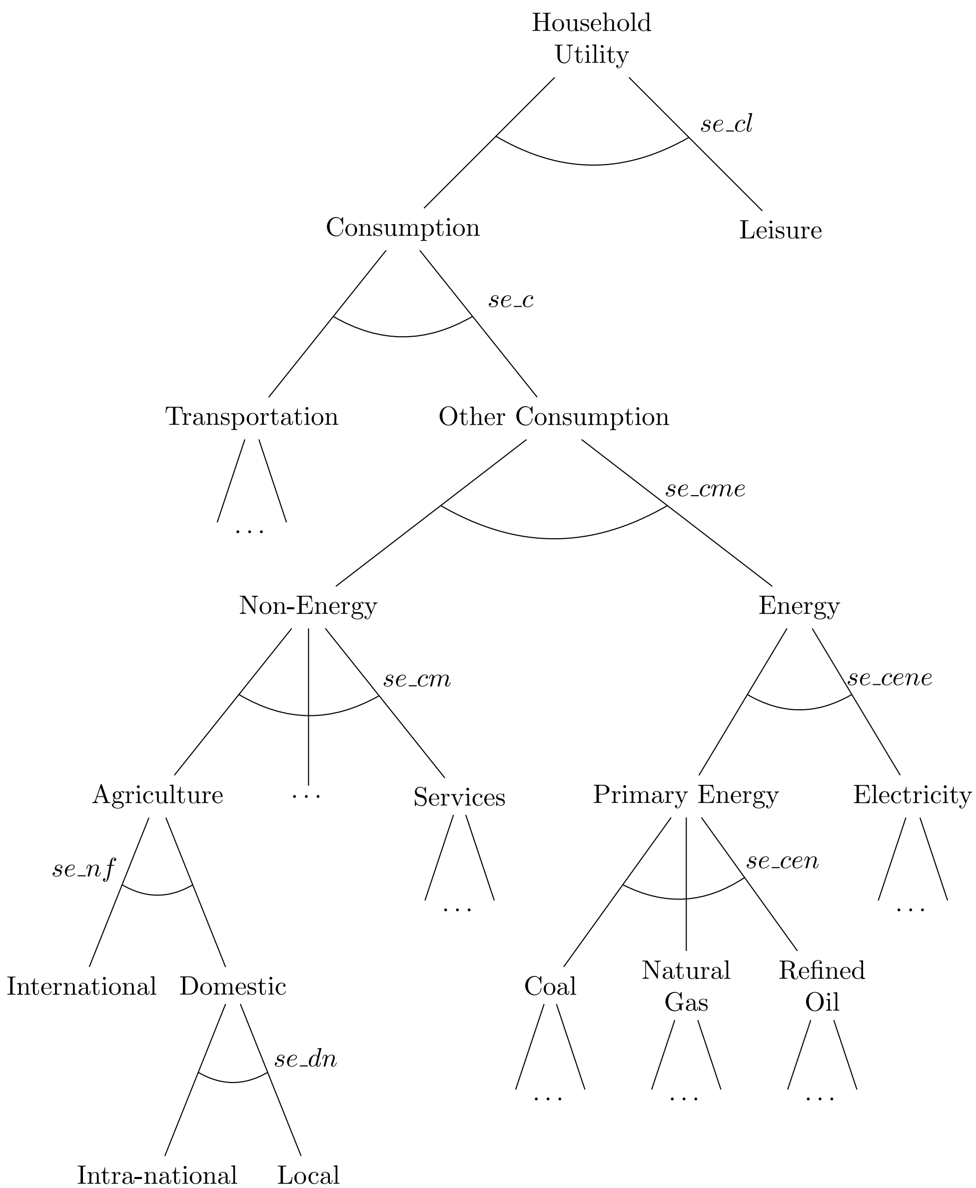

Figure 6: Household Consumption

energy composite

$$cene_{t,r,h} = cene0_{r,h} \left[cs\_cene_{r,h} \left(\frac{cen_{t,r,h}}{cen0_{t,r,h}} \right)^{\frac{se\_cene-1}{se\_cene}} + (1 - cs\_cene_{r,h}) \left(\frac{cd_{t,r,ele,h}}{cd0_{t,r,ele,h}} \right)^{\frac{se\_cene-1}{se\_cene}} \right]^{\frac{se\_cene}{se\_cene-1}}, \tag{29}$$

and $cen_{t,r,s}$ is the primary energy composite

$$\begin{aligned} cen_{t,r,h} = cen0_{r,h} \Bigg[& cs\_cen_{r,h,col} \left(\frac{cd_{t,r,col,h}}{cd0_{t,r,col,h}} \right)^{\frac{se\_cen-1}{se\_cen}} + cs\_cen_{r,h,ref} \left(\frac{cd_{t,r,ref,h}}{cd0_{t,r,ref,h}} \right)^{\frac{se\_cen-1}{se\_cen}} \\ & + cs\_cen_{r,h,gas} \left(\frac{cd_{t,r,gas,h}}{cd0_{t,r,gas,h}} \right)^{\frac{se\_cen-1}{se\_cen}} \Bigg]^{\frac{se\_cen}{se\_cen-1}}. \end{aligned} \tag{30}$$

Since households are assumed to "purchase" leisure at its opportunity cost (i.e., the wage rate), the household labor supply, $l_{t,r,h}$, will be determined according to the time endowment constraint

$$te_{t,r,h} = leis_{t,r,h} + l_{t,r,h}. \tag{31}$$

2.5 Government and Taxes

There is a single national government that imposes ad valorem taxes on labor, capital, production, and consumption, $tl_{t,r}$, $tk_{t,r}$, $ty_{t,r,s}$, and $tc_{t,r}$, respectively. The taxes are region specific, the production tax is sector specific as well, and while they remain constant over time in the baseline we allow for the possibility of future changes in policy simulations.

Government purchases in region r are assumed to be Leontief, such that

$$gov_{t,r} = gov0_{t,r} \min_s \left(\frac{g_{t,r,s}}{g0_{r,s}} \right), \tag{32}$$

where $g_{t,r,s}$ is public demand for commodity s in region r, and $gov_{t,r}$ is the composite public consumption good. The government is assumed to keep real government expenditures per effective household in a region fixed, such that

$$gov_{t,r} = gov0_{t,r} \frac{\sum_h n_{t,r,h}}{\sum_h n0_{r,h}}. \tag{33}$$

The government's budget constraint is

$$\begin{aligned} \sum_r pgov_{t,r} gov_{t,r} + \sum_h tran_{t,r,h} = & \sum_r \sum_s ty_{t,r,s} \left(y_{t,r,s} + y\_ex_{t,r,s} \right) + tk_{t,r} \left(kd_{t,r,s} + kd\_ex_{t,r,s} + res_{t,r,s} \right) \\ & + tl_{t,r} ld_{t,r,s} + \sum_h tc_{t,r} cd_{t,r,s,h}. \end{aligned} \tag{34}$$

The government's budget is balanced through lump sum transfers $incadj_t$, which are shared out to household's based on their share of national consumption in the benchmark dataset. Therefore, net transfers to households are

$$tran_{t,r,h} = tran0_{r,h} \frac{n_{t,r,h}}{n0_{r,h}} + incadj_t \frac{c_{t,r,h}}{\sum_{r,h} c_{t,r,h}}, \tag{35}$$

such that other real transfer payments per capita remain constant.

2.6 Market Clearance

Given firm, household, and government behavior and the capital dynamics described in the preceding sections, prices in equilibrium are assumed to clear all markets and eliminate any intertemporal arbitrage opportunities.

The price of the Armington aggregate, $pa_{t,r,s}$, clears the goods market

$$a_{t,r,s} = \sum_{ss} id_{t,r,s,ss} + id\_ex_{t,r,s,ss} + \sum_{h} cd_{t,r,s,h} + i_{t,r,s} + g_{t,r,s}. \tag{36}$$

The price of domestic output consumed domestically, $pd_{t,r,s}$, clears the domestic market

$$\frac{y\_ex_{t,r,s} + y_{t,r,s}}{y\_ex0_{r,s} + y0_{r,s}} \left(\frac{pd_{t,r,s}}{py_{t,r,s}} \right)^{te\_dx} = \frac{d_{t,r,s}}{d0_{r,s}}, \tag{37}$$

where the left hand side defines the optimal share of output supplied to the domestic market based on the output transformation function in (2). The price of labor, $pl_{t,r}$, (i.e., the wage rate) clears the labor market

$$\sum_{h} l_{t,r,h} = \sum_{s} ld_{t,r,s} + ld\_ex_{t,r,s}. \tag{38}$$

The rental rate for sector specific extant capital, $pr\_ex_{t,r,s}$, clears the market for extant capital

$$\frac{k\_ex_{t,r}}{k\_ex0_{r}} \left(\frac{pr\_ex_{t,r,s}}{pr\_ex\_agg_{t,r}} \right)^{te\_k\_ex} = \frac{kd\_ex_{t,r,s}}{kd\_ex0_{r,s}}, \tag{39}$$

where the left hand side defines the optimal share of extant capital supplied to sector s based on the extant transformation function in (16). The rental rate for new capital, $pr_{t,r}$, clears the market for new capital

$$k_{t,r} = \sum_{s} kd_{t,r,s}. \tag{40}$$

The price of new capital, $pk_{t,r}$, clears the investment market

$$k_{t-1,r} \left(1 - \delta\right) + inv_{t-1,r} = k_{t,r}. \tag{41}$$

The price of foreign exchange, pfx_{t}, clears the foreign exchange market

$$\sum_{r,s} x_{t,r,s,ftrd} + \sum_{r,h} bopdef_{t,r,h} = \sum_{r,s} m_{t,r,s,ftrd}. \tag{42}$$

The price of commodities on the national market, $pn_{t,s}$, clears the market for national trade

$$\sum_{r} x_{t,r,s,dtrd} = \sum_{r} m_{t,r,s,dtrd}. \tag{43}$$

The rental rate for sector specific fixed factors, $pres_{t,r,s}$, clears the market for sector specific fixed factors

$$\sum_{h} resh_{t,r,s} = res_{t,r,s}. \tag{44}$$

Given that the CES and CET functions defining much of the model's structure are homothetic, the prices for composite goods (e.g., $py_{t,r,s}$ and $pcl_{t,r,h}$) are defined by their unit cost.

3 Calibration

There are multiple sets of data and parameters that define the calibration of the model. The benchmark social accounting matrix; the substitution and transformation elasticities in the model's production and utility functions; parameters defining the transformation and depreciation of capital stocks; tax rates; and the parameters defining the baseline projection. This section describes the sources of each of these in turn.

3.1 Benchmark Data

The benchmark data is based on IMPLAN's 2016 database of the U.S. economy aggregated up to the 23 sectors in Table 1 for each of the nine regions in Figure 1, five representative households in Table 3, and the single government.[4] The data is used to define the benchmark year values and cost shares. In this section we describe transformations and modifications made to the database to conform to the structure of our model. Smaller transformations include:

- Household exports, which are primarily purchases by foreign tourists, are shared out across commodities based on final good consumption shares and transfered from households to sector foreign exports.
- Government production (make and use) are integrated with private sector production.
- Investment demand, $i0_{r,s}$, is determined as the residual that would lead the goods market clearance condition in (36) to hold.
- Regional balance of payments are shared out to households based on their share of regional final goods consumption,

$$bopdef0_{r,h} = \sum_{s} m0_{r,s,ftrd} - x0_{r,s,ftrd} \frac{\sum_{s} cd0_{r,s,h}}{\sum_{s,h} cd0_{r,s,h}}. \tag{45}$$

3.1.1 Crude Oil and Natural Gas Extraction Disaggregation

The underlying IMPLAN data does not distinguish between crude oil and natural gas extraction. Therefore, we disaggregate the single IMPLAN oil and gas extraction sector into separate natural gas extraction and crude oil extraction sectors. To determine the natural gas share of consumption/use we assume that crude oil serves as an intermediate input only to the petroleum refining sector and intermediate inputs to all other sectors are only natural gas, and make the same assumptions for household and government consumption and investment demand. In the IMPLAN data, some of the intermediate inputs to the petroleum refining sector are natural gas. To determine that share and the natural gas share of production and trade we minimize the sum of squared deviations for those shares from observed values or assumed shares conditional on market clearance conditions and the assumption of weakly positive domestic use of production. The observed or assumed shares we try to match are derived as follows:

1. The observed share of natural gas production by region is defined using EIA data on crude oil and natural gas production by state aggregated up to the regional level. To arrive at a value share we multiply state level production quantities by EIA data on state level wellhead prices for crude oil and city gate natural gas prices as a proxy for natural gas well head prices.

[4] IMPLAN Group, LLC, 16740 Birkdale Commons Parkway, Suite 206, Huntersville, NC 28078 www.IMPLAN.com

2. The shares of natural gas international imports and exports by region are defined using census data on state level international imports and exports of crude oil and natural gas aggregated to the regional level.
3. A region's intra-national import share of natural gas is assumed to be similar to the region's share of natural gas use relative to the crude oil and natural gas total. A region's intra-national export share of natural gas is assumed to be similar to the share of natural gas production in the region.
4. The observed share of natural gas used as an intermediate input in the refining sector is estimated based on national annual averages of crude oil and natural gas inputs to the sector collected by EIA and converted to values using the Brent and Henry Hub average annual prices as reported by EIA.

3.1.2 Filtering and Balancing Benchmark

To improve the computational performance of the model we filter out small values and rebalance the SAM. We remove any value less than $.5 \times 10^{-4}$ and any intermediate input whose cost share is less than $.5 \times 10^{-4}$.

After filtering small values the SAM is rebalanaced by minimizing the squared percent deviation from the original values weighted by the original values. Specifically we solve for new values of intermediate input demand, $id0_{r,ss,s}$, labor demand, $ld0_{r,s}$, capital demand, $kd0_{r,s}$, imports, $m0_{r,s,trd}$, exports, $x0_{r,s,trd}$, household consumption, $cd0_{r,s,h}$, government spending, $g0_{r,s}$, investment, $i0_{r,s}$, capital endowment, labor endowment, household savings, and inter-regional lump sum government transfers, $incadj$. This optimization is subject to the market clearance conditions in (36), (38), (40), and (114), the budget constraints in (22) and (34), the balance of payment sharing in (45), zero profit condition

$$(1 - ty_{r,s})y0_{r,s} = \sum_{s} id0_{r,ss,s} + (1 + tl_r)\, ld0_{r,s} + (1 + tk_r)\, kd0_{r,s}, \tag{46}$$

regional investment equals household savings

$$\sum_{s} i0_{r,s} = \sum_{h} kh0_{t+1,r,h} - kh0_{t,r,h} \tag{47}$$

and weakly positive domestic own use

$$y0_{t,r,s} > \sum_{trd} x0_{r,s,trd}. \tag{48}$$

The balancing occurs prior to any distinction being made between types of capital: new, extant, and that of fixed factor resources, as covered in the next section. Therefore, the notation is somewhat simpler.

3.1.3 Extant Capital and Fixed Factors

Returns to fixed factors are removed from the capital returns in the IMPLAN database. It is assumed that 25% of the capital demand in the resource extraction sectors and 40% of capital demand in the agriculture and forestry sectors are associated with fixed factors.

We calculate the share of capital in the benchmark year representing new capital as the level of capital that would have been newly formed in that year along the balanced growth path, such that

$$ex\_share_r = \frac{\frac{inv0_{t,r}}{1+\gamma+\omega}}{k0_r}, \tag{49}$$

Table 4: Tax/Subsidy Rates on Production

	nen	mat	enc	wnc	sat	esc	wsc	mnt	pac
agf	0.03	0.02	0.01	0.01	0.00	0.00	0.01	0.00	0.00
cru	0.00	0.05	0.03	0.19	0.03	0.09	0.12	0.15	0.18
col	0.00	0.03	0.07	0.24	0.08	0.05	0.07	0.10	0.10
min	0.03	0.02	0.02	0.04	0.03	0.02	0.02	0.04	0.02
ele	0.08	0.10	0.09	0.06	0.10	0.05	0.07	0.07	0.09
gas	0.03	0.05	0.05	0.08	0.05	0.05	0.11	0.11	0.13
wsu	-0.07	-0.10	-0.06	-0.02	-0.02	-0.01	0.00	-0.01	-0.03
con	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
fbm	0.02	0.05	0.01	0.01	0.05	0.04	0.01	0.01	0.03
wpm	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ref	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
chm	0.02	0.02	0.02	0.01	0.01	0.02	0.01	0.01	0.02
prm	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
cem	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
pmm	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
fmm	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
cpu	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
tem	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00
bom	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01
trn	0.02	0.03	0.04	0.03	0.04	0.02	0.03	0.05	0.04
ttn	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01
srv	0.04	0.05	0.05	0.05	0.05	0.06	0.06	0.05	0.05
hlt	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

where γ is the rate of population growth.

3.2 Taxes

As previously noted, the model explicitly includes consumption, tc_r, labor, tl_r, capital, tk_r, and other business taxes/subsidies, $ty_{r,s}$. Except for the capital tax rate, the taxes are introduced into the dataset prior to aggregation to the model's regions. When aggregating the dataset, taxes are set to keep the tax revenue constant between the disaggregated and aggregated datasets. Production taxes net of any subsidies, $ty_{r,s}$, are based on the average rate observed in the IMPLAN database. The production tax rates are presented in Table 4.

The capital tax, tk_r, is assumed to be constant across the U.S. and is based on Paltsev et al. (2005). Labor tax rates, tl_r by state are the sum of the payroll taxes, calculated as the average FICA tax rate observed in the IMPLAN dataset, along with the average effective marginal wage tax rates in NBER's Taxsim database (Feenberg and Coutts, 1993). State level consumption taxes are based on estimates of the combined local and state consumption tax rates from the Tax Foundation.[5] The tax rates on labor, capital, and consumption are presented in Table 5.

3.3 Substitution Elasticities

As is common with applied CGE models the input-output data is used to define the benchmark value shares and the free parameters are defined by the substitution elasticity parameters. The list of substi-

[5] `http://taxfoundation.org/article/state-and-local-sales-tax-rates-2011-2013`

Table 5: Tax Rates on Capital, Labor, and Consumption

Region	tk	tl	tc
nen	0.38	0.40	0.06
mat	0.38	0.40	0.07
enc	0.38	0.37	0.07
wnc	0.38	0.39	0.07
sat	0.38	0.38	0.06
esc	0.38	0.36	0.08
wsc	0.38	0.35	0.08
mnt	0.38	0.37	0.07
pac	0.38	0.40	0.08

tution elasticities included in the model is presented in Table 6.

3.3.1 Armington Elasticities

The sector-specific Armington elasticities between national and foreign goods, se_nf, are based on the estimates included in the GTAP database (Hertel et al., 2008). The GTAP elasticities are based on econometrically estimated substitution elasticities between imports across foreign sources, se_m, by Hertel et al. (2007) and the "rule of two." The rule, first proposed by Jomini et al. (1991) and applied widely in CGE modeling, suggests that elasticity of substitution across foreign sources is twice as large as the elasticity of substitution between domestic and imported commodities.[6] Such that,

$$se\_nf = \frac{se\_m}{2}. \tag{50}$$

In cases where more than one of the 57 GTAP sectors map into one of the sectors in SAGE, we use value weighted averages based on imports by the U.S. at world prices.

To define the elasticity of substitution between domestic goods and intra-national imports we follow the work of Caron and Rausch (2013). They provide a framework for estimating U.S. intra-national trade elasticities of substitution based on empirical estimates of international and domestic border effects. Specifically, they note that the relative strength of the intra-national and international border effects, α, is defined by the ratio of one minus the substitution elasticities between intra-national sources, se_d, and international sources, se_m, such that

$$\alpha = \frac{1 - se\_d}{1 - se\_m}. \tag{51}$$

Given an estimate for α and se_m, this relationship may be used to solve for the substitution elasticity across domestic sources, se_d. We follow Caron and Rausch (2013) and apply the rule of two to calibrate the substitution elasticity between locally produced goods in the region and intra-national imports, such that $se\_dn = se\_d/2$. Given this relationship, along with (50) and (51), we can solve for the substitution elasticity between locally produced goods as

$$se\_dn = \frac{1}{2} - \alpha \left(\frac{1}{2} - se\_nf \right). \tag{52}$$

Coughlin and Novy (2013) estimate both intra-national and international border effects for the U.S., which may then be used to calculate a value for α. Based on their results we assume that α is 1.868.

[6] Using a back-casting experiment, Liu et al. (2004) recently found no evidence to reject the rule of two, providing additional support for its continued use.

Table 6: Elasticity Parameters

Parameter	Description
Standard Production	
se_klem	Substitution elasticity between material inputs and energy-value-added
se_kle	Substitution elasticity between energy and value added
se_kl	Substitution elasticity between capital and labor
se_ene	Substitution elasticity between electricity and primary energy
se_en	Substitution elasticity among primary energy sources
Agriculture and Forestry Specific	
se_lemkl	Substitution elasticity between land-materials-energy and value-added
se_lem	Substitution elasticity between land and materials-energy
se_em	Substitution elasticity between energy and materials
Resource Extraction Specific	
se_rklem	Substitution elasticity between resource and materials-energy-value-added
Trade	
se_nf	Armington elasticity of substitution between domestic aggregate and foreign goods
se_dn	Armington elasticity of substitution between domestic and national imports
te_dx	Transformation elasticity between domestic and exported goods
Putty-Clay Capital	
te_k_ex	Transformation elasticity of sector differentiated extant capital
Household	
se_cl	Substitution elasticity between consumption bundle and leisure
se_c	Substitution elasticity between transportation and other consumption goods
se_cme	Substitution elasticity between energy and other non-transportation consumption goods
se_cm	Substitution elasticity between non-transportation/non-energy consumption goods
se_cen	Substitution elasticity between primary energy consumption
se_cene	Substitution elasticity between electricity and primary energy consumption
eta	Inverse intertemporal substitution elasticity of consumption

Table 7: SAGE Elasticities

Sector	*se_kl*	*se_kle*	*se_klem*	*se_ene*	*se_en*	*se_nf*	*se_dn*
agf	1.07	0.40	0.98	0.61	0.33	2.45	4.13
col	0.79	0.42	0.22	0.61	0.33	3.05	5.26
cru	0.79	0.42	0.22	0.61	0.33	7.30	13.20
gas	0.79	0.42	0.22	0.61	0.33	2.80	4.80
min	0.79	0.42	0.22	0.61	0.33	0.90	1.25
fbm	0.22	0.19	0.63	0.61	0.33	2.66	4.53
bom	0.56	0.19	0.56	0.61	0.33	4.01	7.06
wpm	0.12	0.24	0.67	0.61	0.33	3.06	5.28
ref	0.73	0.38	0.42	0.61	0.33	2.10	3.49
chm	0.24	0.72	0.94	0.61	0.33	3.30	5.73
prm	0.12	0.18	0.68	0.61	0.33	3.30	5.73
cem	0.20	0.25	0.81	0.61	0.33	2.90	4.98
pmm	0.18	1.01	0.11	0.61	0.33	3.74	6.56
fmm	0.18	1.01	0.11	0.61	0.33	3.75	6.57
tem	0.18	0.16	0.38	0.61	0.33	3.46	6.02
cpu	0.10	1.06	0.64	0.61	0.33	4.40	7.79
ele	1.00	0.46	0.68	0.01	0.30	2.80	4.80
wsu	1.00	0.46	0.68	0.61	0.33	2.80	4.80
con	0.17	0.15	0.61	0.61	0.33	1.90	3.12
trn	0.54	0.46	0.73	0.25	0.25	1.90	3.12
ttn	0.14	0.42	0.22	0.25	0.25	1.90	3.12
srv	0.32	0.27	0.66	0.53	0.24	1.90	3.12
hlt	0.58	0.16	0.80	0.53	0.24	1.90	3.12

The SAGE values for *se_nf* and *se_dn* are presented in Table 7.

We also follow Caron and Rausch (2013) is setting the transformation elasticity of output between domestic use, national exports, and international exports, *te_dx*, to 2.

3.3.2 Production Elasticities of Substitution

Koesler and Schymura (2015) provide empirical estimates of the capital-labor substitution elasticities (*se_kl*), (capital-labor)-energy substitution elasticities (*se_kle*), and (capital-labor-energy)-materials substitution elasticities (*se_klem*) at the industry level using a CES nesting structure that is consistent with our standard production structure in Figure 3 and the resource extraction sectors production structure in Figure 4. The estimates are calculated with a consistent approach and dataset, the panel nature of their dataset allows for the estimation of long-run elasticities, and they have been previously applied to CGE modeling (e.g., Böhringer et al. (2016)). For cases where a one-to-one mapping between their sectors and SAGE's sectors is not possible we use a weighted average of the Koesler and Schymura (2015) elasticities, where the weighting is by the U.S. sectoral output value in the last year of their dataset. For five sectors, the estimation routine of Koesler and Schymura (2015) returned non-finite values for *se_kl*. In these cases we use values from the recent study by Young (2013), which estimates value-added substitution elasticities for the U.S. For service sectors where there is not a direct mapping between the two studies we use the value from Young (2013) for their aggregated services sector.[7] Koesler and Schymura (2015) also reported a non-finite value for *se_kle* in the refining sector. In this case we apply the total industry value. The SAGE values for *se_kl*, *se_kle*, and *se_klem* are presented in Table 7.

[7] We use the non-normalized generalized method of moments estimates from Young (2013).

For the interfuel substitution elasticities we use estimates from Serletis et al. (2010a), which provide the most recent estimates for the U.S. based on contemporary data and disaggregated across the industrial, commercial, electricity, and residential sectors, of which we are aware. For the primary energy substitution elasticity, se_en, we use a weighted average of the Morishima elasticity across refined oil, coal, and natural gas. The weights represent the sectoral expenditures on the two fuels included in the Morishima elasticity in the last year of the sample in Serletis et al. (2010a) based on EIA's Annual Energy Review. We use a similar approach for the calibrating the substitution elasticity between the primary energy composite and electricity, se_ene, using the Morishima elasticity estimates for electricity and primary fuels from Serletis et al. (2010a). We assign values from the industrial sector to the manufacturing and resource extraction sectors in the model. We assign values from the commercial sector to the services and healthcare sectors (srv and hlt). The electricity sector estimates are assigned to the electricity sector (ele). For the transportation sectors we base the substitution elasticities on the estimates of Serletis et al. (2010b) for high-income countries. The SAGE values for se_en and se_ene are presented in Table 7.

3.3.3 Resource Extraction

In sectors with a fixed factor input, including the resource extraction sectors and the agriculture and forestry sectors, the elasticity of substitution between the fixed factor resource and other inputs, se_rklem, is calibrated over the model horizon to match a path of short- to long-run supply elasticities following the approach of Balistreri and Rutherford (2001). In partial equilibrium with fixed prices for all non-resource inputs and a fixed quantity for the resource, the elasticity of supply for a given sector is given by

$$\eta = -\sigma_{res}, \tag{53}$$

where σ_{res} is the Allen-Uzawa own-price elasticity of substitution (Hertel and Tsigas, 2002). In the nesting structure for sectors with a fixed factor, as depicted in Figure 4, the Allen-Uzawa own price elasticity for sector s in region r is

$$\sigma_{res} = -se\_rklem_{r,s}\left(\theta_{r,s,res}^{-1} - 1\right), \tag{54}$$

where $\theta_{r,s,res}$ is the benchmark resource cost share of total costs (Keller, 1976). Combining (53) and (54) provides the calibrated substitution elasticity for a given elasticity of supply

$$se\_rklem_{r,s} = \frac{\eta}{\theta_{r,s,res}^{-1} - 1}. \tag{55}$$

In the initial model year the elasticity of supply for sector s is calibrated to a sector specific short-run supply elasticity, $\eta_{s,sr}$, and converges to a long-run supply elasticity, $\eta_{s,lr}$, at the rate ρ_s. For model year t and sector s, this leads to a substitution elasticity of

$$se\_rklem_{r,s,t} = \frac{1}{\theta_{r,s,res}^{-1} - 1}\left[\eta_{s,lr} - \left(\eta_{s,lr} - \eta_{s,sr}\right)e^{-\rho_s t}\right]. \tag{56}$$

Arora (2014) examines natural gas supply elasticities in the U.S. before and after the expansion of shale gas production through hydraulic fracturing, finding evidence of more elastic supply in recent years. Based on these estimates, Arora and Cai (2014) suggest a short-run supply elasticity of 0.02 and long-run supply elasticity of 0.5 for natural gas production for a reference case in CGE modeling. We follow this approach and apply a rate of convergence to the long-run elasticity of 0.15 to be reflective of the rate of adjustment observed by Arora (2014).

Table 8: Fixed Factor Sector Elasticities of Supply

Sector	Short-Run	Long-Run	Convergence Rate
col	0.40	1.90	0.07
cru	0.05	0.25	0.15
gas	0.02	0.50	0.15
min	2.50	2.50	∞
agf	1.19	1.19	∞

U.S. oil supply is also considered to be inelastic. Huntington (1992) reviewed expectations of U.S. crude oil supply elasticities through the inferred elasticities in commonly used energy modeling systems of the time and found an average short-run elasticity of 0.05 and an average long-run elasticity of 0.40. There is evidence that in the decades since those models were calibrated, the oil supply has become more inelastic (Greene and Liu, 2015). For example, Krichene (2002) estimated the long-run crude oil supply elasticity to be 0.25 over the period 1918-1999, with lower elasticity estimates when the sample was restricted to the later years. A long-run supply elasticity in the range of 0.25 to 0.40 would be consistent with inferred U.S. long-run supply elasticities in the recent U.S. Annual Energy Outlook based on the high and low oil price scenarios (EIA, 2018). Using a long-run supply elasticity of 0.25, Beckman et al. (2011) found the GTAP-E model was able to adequately capture the variance of oil price responses to supply and demand shocks based on historical observations. Based on these lines of evidence, we apply a short-run crude oil supply elasticity of 0.05 and a long-run supply elasticity of 0.25, with a rate of convergence of 0.15.

The supply of coal in the U.S. is generally thought to be elastic. We follow the approach of Balistreri and Rutherford (2001) and set the short-run supply elasticity of coal to 0.4 and the long-run supply elasticity to 1.9, with a convergence elasticity of 0.07. A long-run supply elasticity of 1.9 is in line with previous modeling exercises (Golombek et al. (1995); Brown and Huntington (2003)).

We follow Hertel and Tsigas (2002) in calibrating the price elasticity of supply in the agriculture and forestry sector, agf, to 1.19. We calibrate the elasticity of supply in the metal ore and nonmetallic mineral mining sector, min, to 5.0. In these sectors we do not distinguish between a short- and long-run supply elasticity.

The short- and long-run supply elasticities and rates of convergence for the sectors associated with fixed factors are presented in Table 8.

3.3.4 Consumption Elasticities

For the elasticity of substitution across the consumption of non-energy goods, se_cm, the elasticity of substitution across consumption of non-energy goods and energy, se_cme, and the elasticity of substitution across non-transportation and transportation goods, se_c, we follow the specification of Rausch et al. (2011). To calibrate the elasticity of substitution across household primary energy consumption and the elasticity of substitution between primary energy, se_cen, and electricity consumption, se_cene, we apply the same approach used in Section 3.3.2 based on the empirical estimates of Serletis et al. (2010a). The parameter defining the intertemporal elasticity of substitution for consumption, η, is set to unity following Jorgenson et al. (2013) based on their empirical estimates. These value are presented in Table 9.

The consumption-leisure substitution elasticity is determined jointly with the time endowment in the model to match observed estimates of the compensated and uncompensated labor supply elasticities in

Table 9: Household Substitution Elasticities

Parameter	Value
se_cl	Calibrated
se_c	1.00
se_cme	0.25
se_cm	0.25
se_cene	0.67
se_cen	0.24
eta	1.00

a static setting. Consider the demand system in (25) and the simplified budget constraint

$$pcl_{t,r,h}cl_{t,r,h} = pl_{t,r}te_{t,r,h} + \pi_{t,r,h}, \tag{57}$$

where $\pi_{t,r,h}$ represents non-labor income. The Marshallian demand for leisure is

$$\begin{aligned} leis_{t,r,h} =& leis0_{r,h} \left(\frac{\pi_{t,r,h} + pl_{t,r}te_{t,r,h}}{\pi 0_{r,h} + pl0_r te0_{r,h}} \right) \left(\frac{pl_{t,r}}{pl0_r} \right)^{-se\_cl} \\ & \times \left[cs\_cl_{r,h} \left(\frac{pc_{t,r,h}}{pc0_{r,h}} \right)^{1-se\_cl} + (1 - cs\_cl_{r,h}) \left(\frac{pl_{t,r}}{pl0_r} \right)^{1-se\_cl} \right]^{-1}. \end{aligned} \tag{58}$$

The uncompensated price elasticity of leisure demand, μ_l, may be obtained from (58), such that

$$\begin{aligned} \mu^{leis}_{t,r,h} \equiv& \frac{\partial leis_{t,r,h}}{\partial pl_{t,r}} \frac{pl_{t,r}}{leis_{t,r,h}} \\ =& \frac{pl_{t,r}te_{t,r,h}}{\pi_{t,r,h} + pl_{t,r}te_{t,r,h}} - (1 - cs\_cl_{r,h}) \left(\frac{pl0_r}{pl_{t,r}} \right)^{se\_cl-1} e\left(pc_{t,r,h}, pl_{t,r}\right)^{1-se\_cl} \\ & + se\_cl \left[(1 - cs\_cl_{r,h}) \left(\frac{pl0_r}{pl_{t,r,h}} \right)^{se\_cl-1} e\left(pc_{t,r,h}, pl_{t,r}\right)^{1-se\_cl} - 1 \right], \end{aligned} \tag{59}$$

where

$$e\left(pc_{t,r,h}, pl_{t,r}\right) = \left[cs\_cl_{r,h} \left(\frac{pc_{t,r,h}}{pc0_{r,h}} \right)^{1-se\_cl} + (1 - cs\_cl_{r,h}) \left(\frac{pl_{t,r}}{pl0_r} \right)^{1-se\_cl} \right]^{\frac{1}{se\_cl-1}}. \tag{60}$$

The first two components of (59) define the income elasticity of leisure,

$$\mu^{I}_{t,r,h} = \frac{pl_{t,r}te_{t,r,h}}{\pi_{t,r,h} + pl_{t,r}te_{t,r,h}} - (1 - cs\_cl_{r,h}) \left(\frac{pl0_r}{pl_{t,r}} \right)^{se\_cl-1} e\left(pc_{t,r,h}, pl_{t,r}\right)^{1-se\_cl}, \tag{61}$$

and the third component represents the substitution effect, or the uncompensated price elasticity of leisure demand,

$$\mu^{leis|\bar{c}l}_{t,r,h} = se\_cl \left[(1 - cs\_cl_{r,h}) \left(\frac{pl0_r}{pl_{t,r,h}} \right)^{se\_cl-1} e\left(pc_{t,r,h}, pl_{t,r}\right)^{1-se\_cl} - 1 \right] \tag{62}$$

This may be verified through the Hicksian demand function via the Slutsky equation. Given the definition of labor supply, $te_{t,r,h} - leis_{t,r,h}$, the compensated labor supply elasticity, or substitution effect, is

$$\epsilon^{l|\bar{c}l}_{t,r,h} = -\mu^{leis|\bar{c}l}_{t,r,h} \frac{leis_{t,r,h}}{te_{t,r,h} - leis_{t,r,h}}. \tag{63}$$

And the uncompensated labor supply elasticity is

$$\epsilon^{l}_{t,r,h} = -\mu^{leis}_{t,r,h} \frac{leis_{t,r,h}}{te_{t,r,h} - leis_{t,r,h}}, \tag{64}$$

which, given (59) and (61), may be written as

$$\epsilon^{l}_{t,r,h} = -\left(\mu^{I}_{t,r,h} + \mu^{l|\bar{cl}}_{t,r,h}\right) \frac{leis_{t,r,h}}{te_{t,r,h} - leis_{t,r,h}}. \tag{65}$$

We define the share of the time endowment spent on leisure as $\phi_{t,r,h} = leis_{t,r,h}/te_{t,r,h}$ and rewrite (63) and (65) as

$$\epsilon^{l|\bar{cl}}_{t,r,h} = \frac{-\phi_{t,r,h}}{1-\phi_{t,r,h}} \mu^{l|\bar{cl}}_{t,r,h} \tag{66}$$

and

$$\epsilon^{l}_{t,r,h} = \frac{-\phi_{t,r,h}}{1-\phi_{t,r,h}} \left(\mu^{I}_{t,r,h} + \mu^{l|\bar{cl}}_{t,r,h}\right). \tag{67}$$

Substituting (66) into (67) yields

$$\epsilon^{l}_{t,r,h} = \frac{-\phi_{t,r,h}}{1-\phi_{t,r,h}} \mu^{I}_{t,r,h} + \epsilon^{l|\bar{cl}}_{t,r,h}. \tag{68}$$

From (62), the benchmark year income elasticity of leisure is

$$\mu^{I}_{0,r,h} = \frac{pl0_r te0_{r,h}}{\pi 0_{r,h} + pl0_r te0_{r,h}} - (1 - cs\_cl_{r,h}) = (1 - cs\_cl_{r,h}) \frac{1-\phi_{0,r,h}}{\phi_{0,r,h}}. \tag{69}$$

Substituting (69) into (68) yields

$$\epsilon^{l}_{0,r,h} = \epsilon^{l|\bar{cl}}_{0,r,h} + cs\_cl_{r,h} - 1. \tag{70}$$

Labor supply elasticity estimates from the literature may be used with (70) to determine a value for $cs\_cl_{r,h}$. Assuming that in the benchmark all prices are normalized to unity, and given (70), the definition of $cs\_cl_{r,h}$, and an estimate of the income elasticity of labor, $\hat{\epsilon}^I$, the calibrated benchmark value of leisure is approximated by

$$leis0_{r,h} = -c0_{r,h} \frac{\hat{\epsilon}^I}{1+\hat{\epsilon}^I}. \tag{71}$$

From (62), the benchmark uncompensated leisure demand elasticity is

$$\mu^{\bar{l}|\bar{cl}}_{0,r,h} = -se\_cl \cdot cs\_cl_{r,h}. \tag{72}$$

Substituting (66) into (72) yields the calibrated version of the elasticity of substitution between consumption and leisure,

$$se\_cl = \left(\frac{te0_{r,h}}{leis0_{r,h}} - 1\right) \frac{\hat{\epsilon}^{l|\bar{cl}}}{1+\hat{\epsilon}^I}, \tag{73}$$

where $\hat{\epsilon}^{l|\bar{cl}}$ is the empirical estimate of the substitution elasticity and the observed labor earnings are combined with the calibrated benchmark value of leisure in (71) to determine the time endowment $te0_{r,h} = l0_{r,h} + leis0_{r,h}$.

To calibrate the time endowment and substitution elasticity between consumption and leisure we use the conclusions from the literature review by McClelland and Mok (2012) on estimates of the income and substitution effects. Specifically, they conclude that estimates on the order of $\hat{\epsilon}^I = -0.05$ and $\hat{\epsilon}^{l|\bar{cl}} = 0.20$ to be representative of the most recent empirical evidence. Given the dynamic nature of the model and the baseline calibration that deviates from the assumptions in the simplified static household problem

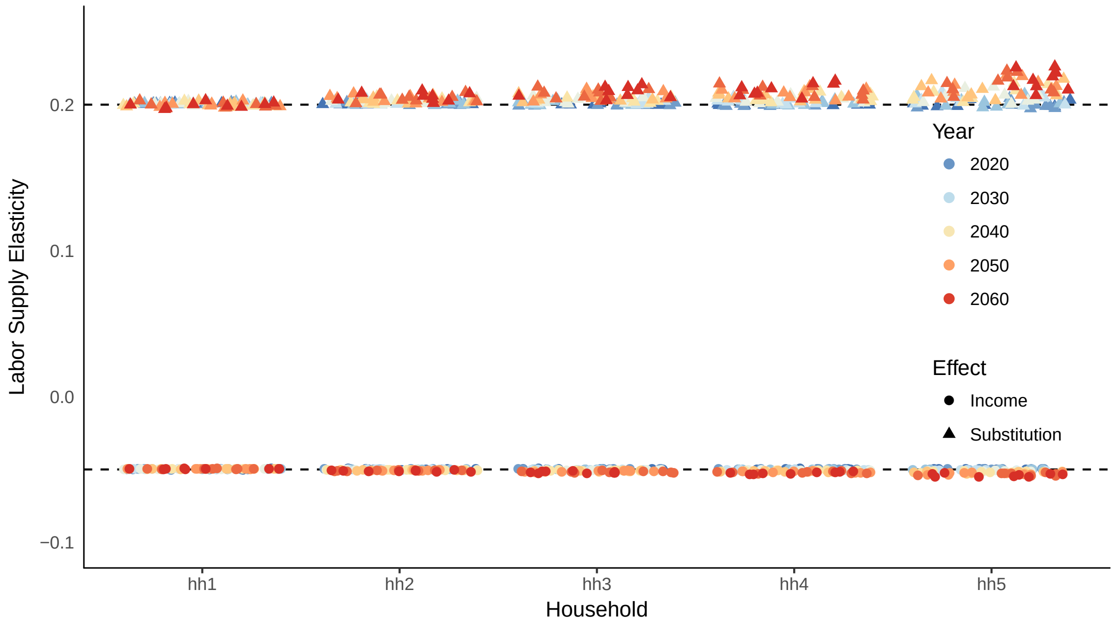

Figure 7: Calibrated Labor Supply Elasticities

above, the model's endogenous labor supply elasticities differ slightly from the calibration points. Figure 7 presents the substitution and income effects implicit in the model's baseline.

3.4 Dynamic Baseline

The reference path for the model is a balanced growth path with population growth and Harrod neutral technological progress. The baseline builds upon the reference path by calibrating the energy demands of future production technologies to be consistent with the U.S. Energy Information Administration's 2018 Annual Energy Outlook (AEO) forecasts.

The steady-state interest rate along the balanced growth path, $rbar$, is set to 0.045. The interest rate reflects the average after-tax rate of return on private capital. Given the capital tax in Section 3.2, the social return on private capital in the model is 0.072, which is consistent with the average pre-tax rate of return on capital observed between 1960 and 2014 (CEA, 2017). The depreciation rate, δ, is set to 0.05, which is the average U.S. capital depreciation rate from 1950 to 2014 as estimated by Feenstra et al. (2015). This rate is applied to both new and extant capital.

Population is assumed to grow at rate γ, such that

$$n_{t,r,h} = n0_{r,h} \left(1 + \gamma\right)^t , \tag{74}$$

where γ is set to the average annual population growth rate in the AEO, 0.006. Technological progress is assumed to be Harrod neutral (labor augmenting). Labor productivity growth, ω, is assumed to be 0.016, which is the average annual labor productivity growth in the AEO. Labor productivity growth is implemented through the effective time endowment, such that

$$te_{t,r,h} = te0_{r,h} \left(1 + \gamma + \omega\right)^t . \tag{75}$$

Based on the isoelastic form of the intra-temporal utility function, the pure rate of time preference, ρ, in (21) is defined as

$$\rho = \frac{1 + rbar}{(1 + \omega)^{\eta}} - 1. \tag{76}$$

3.4.1 Baseline Energy Use

We calibrate the cost shares in the production functions to capture expected technological change in the energy efficiency of production, based on the AEO forecasts. To get the unit energy consumption (UEC) we divide the total energy consumption in the AEO by the real value of shipments for the sectors. The National Energy Modeling System (NEMS) used for the AEO has limited ability for fuel switching within or output response within the industrial sectors, so changes in the UEC over time predominately represent exogenous forecasts regarding technological change in energy efficiency. We use the average growth rate of the UEC over the AEO time horizon, denoted as ene_growth_s, to calibrate the cost shares in the production function.

The change in energy efficiency is assumed to be capital embodied. Therefore, the change is represented as a shift from energy use to capital, such that the benchmark values for intermediate, capital, and labor inputs, as well as the cost shares, are time dependent. Such that,

$$id0_{t,r,ss,s} = (1 - ene\_growth_s)^t \, id0_{0,r,ss,s} \;\; ss \in (col, gas, ref, ele) \tag{77}$$

and

$$kd0_{t,r,s} = kd0_{0,r,s} + \left[1 - (1 - ene\_growth_s)^t\right] \sum_{ss \in sene} id0_{0,r,ss,s}, \tag{78}$$

where $sene$ is the set of primary energy commodities plus electricity and the relevant cost shares, cs_kle and cs_kl are updated as well.

The mapping from the AEO sectors to the SAGE sectors, along with the UEC growth parameters are presented in Table 10. For the non-truck transportation sector, trn, the UEC growth rate is based on the average growth rate of fuel efficiency of air transportation as forecast by the AEO, as this represents a large share of the energy consumption for the sector. For the truck transportation sector, ttn, the UEC growth rate is based on the average growth rate of fuel efficiency of truck freight transportation forecast by the AEO. No changes in the energy efficiency of the electricity sector are assumed.

Household and government energy consumption shares are assumed to change over time to match the energy efficiency forecasts in AEO. Consumption shares of electricity, natural gas are assumed to grow at the same average rate as in the AEO forecast, $cd\_ene\_growth_{ele}$ and $cd\_ene\_growth_{gas}$, respectively. The consumption share of refined petroleum is assumed to grow based on the average growth rate of light duty vehicle energy intensity in the AEO forecast, $cd\_ene\_growth_{ref}$.

$$cd0_{t,r,h,s} = (1 - cd\_ene\_growth_s)^t cd0_{0,r,h,s} \;\; s \in (col, gas, ref, ele) \tag{79}$$

and

$$cd0_{t,r,h,s} = cd0_{0,r,h,s} + \left\{ \sum_{ss \in sene} \left[1 - (1 - cd\_ene\_growth_{ss})^t\right] cd0_{0,r,h,ss} \right\} \frac{cd0_{0,r,h,s}}{\sum_{ss \notin sene} cd0_{0,r,h,ss}} \;\; s \notin sene \tag{80}$$

where the relevant cost shares are cs_cen, cs_cene, cs_cme, and cs_c. The values for the growth rates are presented in Table 11.

The share of electricity generated from natural gas in the baseline is roughly consistent with forecasts

Table 10: Unit Energy Consumption Growth Rates

SAGE Sector	AEO Sectors	*ene_growth*
agf	agg	-0.0031
col	ming	-0.0025
min	ming	-0.0025
ele		
gas	ming	-0.0025
cru	ming	-0.0025
wsu	bmf	-0.0107
con	cns	-0.0013
fbm	fdp	-0.0034
wpm	ppm, wdp	-1e-04
ref	ref	-0.0018
chm	bch	-0.004
prm	pli	-0.0142
cem	cem	-0.0081
pmm	ism, aap	-0.0042
fmm	fbp	-0.0135
cpu	cmpr, eei	-0.0111
tem	teq	-0.0131
bom	bmf, ggr, mchi	-0.0133
trn		-0.0062
ttn		-0.0095
srv	comm	-0.0152
hlt	comm	-0.0152

Table 11: Energy Consumption Share Growth Rates

Commodity	*cd_ene_growth*
ele	-0.0191
gas	-0.0206
ref	-0.0157
col	0

from AEO. However, the share of electricity generated from coal, absent of any adjustment, would be higher than AEO forecasts due to regulatory and technological changes favoring renewable sources of generation. Therefore, we adjust the cost share of coal in electricity production by the average growth rate in the share of electricity generated from coal in the AEO forecast, col_ele_growth. The changes are assumed to be picked up by capital and labor, which would be associated with the alternative sources of generation. Specifically, the intermediate, capital, and labor inputs are adjusted over time, such that

$$id0_{t,r,col,ele} = (1 + col\_ele\_growth)^t \, id0_{0,r,col,ele}, \tag{81}$$

$$kd0_{t,r,ele} = kd0_{0,r,ele} + \left[1 - (1 + col\_ele\_growth)^t \, id0_{0,r,col,ele}\right] \frac{kd0_{0,r,ele}}{kl0_{r,ele}}, \tag{82}$$

and

$$ld0_{t,r,ele} = ld0_{0,r,ele} + \left[1 - (1 + col\_ele\_growth)^t \, id0_{0,r,col,ele}\right] \frac{ld0_{0,r,ele}}{kl0_{r,ele}}, \tag{83}$$

where the cost shares cs_en, cs_ene, and cs_kle are also adjusted accordingly.

4 Solution

To solve the model, the primal version of the problem in Section 2 is converted to a mixed complementarity problem (MCP) following Mathiesen (1985) and Rutherford (1999). Embedded in the MCP are the conditions that define profit maximizing firm behavior, welfare maximizing household behavior, market clearance, balanced budgets, and perfect competition.

Given the assumption of constant returns to scale, one can solve for the constant unit cost function of producing good z denoted as $C^z_{t,r,z}$. Perfect competition may then be represented along with profit maximization by zero-profit conditions that assume the unit cost function under optimal behavior is at least as great as the price for the good. If it is the case that the unit cost function is greater than the price, such that profits are negative, it must be the case that the quantity produced is zero, providing the complementarity condition. This will hold for production with both new and extant capital and provision of the Armington aggregate, government goods, and investment. The zero-profit conditions associated with these activities are

$$C^y_{t,r,s}\left(pa_{t,r,1}, \ldots, pa_{t,r,S}, pr_{t,r}, pres_{t,r,s}, pl_{t,r}, tk_{t,r}, tl_{t,r}, ty_{t,r,s}\right) \geq py_{t,r,i} \quad \perp \quad y_{t,r,i} \geq 0, \tag{84}$$

$$C^{y\_ex}_{t,r,i}\left(pa_{t,r,1}, \ldots, pa_{t,r,N}, pr\_ex_{t,r,s}, pl_{t,r}, tk_{t,r}, tl_{t,r}, ty_{t,r,s}\right) \geq py_{t,r,i} \quad \perp \quad y\_ex_{t,r,i} \geq 0, \tag{85}$$

$$C^a_{t,r,i}\left(pd_{t,r,i}, pn_{t,i}, pfx_t\right) \geq pa_{t,r,i} \quad \perp \quad a_{t,r,i} \geq 0, \tag{86}$$

$$C^g_{t,r}\left(pa_{t,r,1}, \ldots, pa_{t,r,S}\right) \geq pgov_{t,r} \quad \perp \quad gov_{t,r} \geq 0, \tag{87}$$

and

$$C^i_{t,r}\left(pa_{t,r,1}, \ldots, pa_{t,r,S}\right) \geq pinv_{t,r} \quad \perp \quad inv_{t,r} \geq 0. \tag{88}$$

where $C^y_{t,r,s}$ is the unit cost function for production of s using new capital based on (3) and (10), $C^{y\_ex}_{t,r,s}$ is the unit cost function for production of s using extant capital based on (13), $C^a_{t,r,s}$ is the unit cost function for the Armington aggregate based on (1), $C^g_{t,r}$ is the unit cost function for the government good based on (32), $C^i_{t,r,s}$ is the unit cost function for the investment good based on (19). A similar condition can be established for the price of full consumption

$$e_{t,r,h}\left(pa_{t,r,1}, \ldots, pa_{t,r,S}, tc_{t,r}\right) \geq pcl_{t,r,h} \quad \perp \quad cl_{t,r,h} \geq 0, \tag{89}$$

where $e_{t,r,h}$ is the unit expenditure function for full consumption based on the inter-temporal preferences in (25). The final zero-profit condition requires that for households to hold capital the price must equal the present value of returns, such that

$$pk_{t,r} \geq pr_{t,r}\left(rbar + \delta\right) + \left(1 - \delta\right) pk_{t+1,r} \quad \perp \quad k_{t,r}. \tag{90}$$

From Shepard's lemma the Hicksian demands for inputs will be the partial derivative of the unit cost function with respect to the price of the input times the level of the activity. Input demands for profit maximizing firms given the equilibrium level of production, such that inputs to production using new capital are

$$id_{t,r,ss,s} = \frac{\partial C^{y}_{t,r,s}}{\partial pa_{t,r,ss}} y_{t,r,s}, \tag{91}$$

$$kd_{t,r,s} = \frac{\partial C^{y}_{t,r,s}}{\partial pr_{t,r}} y_{t,r,s}, \tag{92}$$

$$ld_{t,r,s} = \frac{\partial C^{y}_{t,r,s}}{\partial pl_{t,r}} y_{t,r,s}, \tag{93}$$

and

$$res_{t,r,s} = \frac{\partial C^{y}_{t,r,s}}{\partial pres_{t,r,s}} y_{t,r,s}. \tag{94}$$

Similarly inputs to production using extant capital are defined as

$$id\_ex_{t,r,ss,s} = \frac{\partial C^{y\_ex}_{t,r,s}}{\partial pa_{t,r,ss}} y\_ex_{t,r,s}, \tag{95}$$

$$kd\_ex_{t,r,s} = \frac{\partial C^{y\_ex}_{t,r,s}}{\partial pr\_ex_{t,r,s}} y\_ex_{t,r,s}, \tag{96}$$

and

$$ld\_ex_{t,r,s} = \frac{\partial C^{y\_ex}_{t,r,s}}{\partial pl_{t,r}} y\_ex_{t,r,s}. \tag{97}$$

Noting that as described in Section 2.3, the model does not have vintaged capital for sectors associated with a fixed factor input. The inputs to the formation of capital and government consumption may be similarly defined as

$$g_{t,r,s} = \frac{\partial C^{g}_{t,r}}{\partial pa_{t,r,s}} gov_{t,r} \tag{98}$$

and

$$i_{t,r,s} = \frac{\partial C^{i}_{t,r}}{\partial pa_{t,r,s}} inv_{t,r}. \tag{99}$$

Given the equilibrium level of full consumption, the demands for final consumption goods are

$$cd_{t,r,h,s} = \frac{\partial e_{t,r,h}}{\partial pa_{t,r,s}} cl_{t,r,h}, \tag{100}$$

where leisure demand can be similarly defined as

$$leis_{t,r,h} = \frac{\partial e_{t,r,h}}{\partial pl_{t,r}} cl_{t,r,h}. \tag{101}$$

Imports and domestically-sourced use are defined conditional on the equilibrium level of the Armington aggregate as

$$d_{t,r,s} = \frac{\partial C^{a}_{t,r,s}}{\partial pd_{t,r,s}} a_{t,r,s}, \tag{102}$$

$$m_{t,r,s,dtrd} = \frac{\partial C^a_{t,r,s}}{\partial pn_{t,r}} a_{t,r,s}, \tag{103}$$

and

$$m_{t,r,s,ftrd} = \frac{\partial C^a_{t,r,s}}{\partial pfx_t} a_{t,r,s}. \tag{104}$$

Exports are determined from the CET function in (2), such that

$$x_{t,r,s,dtrd} = \frac{y\_ex_{t,r,s} + y_{t,r,s}}{y\_ex0_{r,s} + y0_{r,s}} \left(\frac{pn_{t,r}}{py_{t,r,s}} \right)^{te\_dx} \tag{105}$$

and

$$x_{t,r,s,ftrd} = \frac{y\_ex_{t,r,s} + y_{t,r,s}}{y\_ex0_{r,s} + y0_{r,s}} \left(\frac{pfx_t}{py_{t,r,s}} \right)^{te\_dx}. \tag{106}$$

Given the Hicksian demands conditional on equilibrium activity levels, the market clearance conditions in Section 2.6 can be defined. If any of the conditions in (36)-(44) holds with strict inequality it would imply that supply exceeds demand in equilibrium, such that the price of that activity's output must be zero. This leads to a series of complementarity conditions, which define the market clearance conditions. The price of the Armington aggregate, $pa_{t,r,s}$, clears the goods market

$$a_{t,r,s} \geq \sum_{ss} id_{t,r,s,ss} + id\_ex_{t,r,s,ss} + \sum_{h} cd_{t,r,s,h} + i_{t,r,s} + g_{t,r,s} \quad \perp \quad pa_{t,r,s} \geq 0 \tag{107}$$

The price of domestic output consumed domestically, $pd_{t,r,s}$, clears the domestic market

$$\frac{y\_ex_{t,r,s} + y_{t,r,s}}{y\_ex0_{r,s} + y0_{r,s}} \left(\frac{pd_{t,r,s}}{py_{t,r,s}} \right)^{te\_dx} \geq \frac{d_{t,r,s}}{d0_{r,s}} \quad \perp \quad pd_{t,r,s} \geq 0, \tag{108}$$

where the left hand side defines the optimal share of output supplied to the domestic market based on the output transformation function in (2). The price of labor, $pl_{t,r}$, (i.e., the wage rate) clears the labor market

$$\sum_{h} l_{t,r,h} \geq \sum_{s} ld_{t,r,s} + ld\_ex_{t,r,s} \quad \perp \quad pl_{t,r} \geq 0 \tag{109}$$

The rental rate for sector specific extant capital, $pr\_ex_{t,r,s}$, clears the market for extant capital

$$\frac{k\_ex_{t,r}}{k\_ex0_r} \left(\frac{pr\_ex_{t,r,s}}{pr\_ex\_agg_{t,r}} \right)^{te\_k\_ex} \geq \frac{kd\_ex_{t,r,s}}{kd\_ex0_{r,s}} \quad \perp \quad pr\_ex_{t,r,s} \geq 0 \tag{110}$$

where the left hand side defines the optimal share of extant capital supplied to sector s based on the extant transformation function in (16). The rental rate for new capital, $pr_{t,r}$, clears the market for new capital

$$k_{t,r} \geq \sum_{s} kd_{t,r,s} \quad \perp \quad pr_{t,r} \geq 0. \tag{111}$$

The price of new capital, $pk_{t,r}$, clears the investment market

$$k_{t-1,r}(1-\delta) + inv_{t-1,r} \geq k_{t,r} \quad \perp \quad pk_{t,r} \geq 0. \tag{112}$$

The price of foreign exchange, pfx_t, clears the foreign exchange market

$$\sum_{r,s} m_{t,r,s,ftrd} \geq \sum_{r,s} x_{t,r,s,ftrd} + \sum_{r,h} bopdef_{t,r,h} \quad \perp \quad pfx_t \geq 0. \tag{113}$$

The price of commodities on the national market, $pn_{t,s}$, clears the market for national trade

$$\sum_r x_{t,r,s,dtrd} \geq \sum_r m_{t,r,s,dtrd} \quad \perp \quad pn_{t,s} \geq 0. \tag{114}$$

The rental rate for sector specific fixed factors, $pres_{t,r,s}$, clears the market for sector specific fixed factors

$$\sum_h resh_{t,r,s} \geq res_{t,r,s} \quad \perp \quad pres_{t,r,s} \geq 0. \tag{115}$$

Equilibrium also requires that aggregate household holdings of capital, $kh_{t,r,h}$, equal the aggregate level of capital in the region, $k_{t,r}$, however due to Walras law one of the constraints is redundant and we choose to omit this capital aggregation constraint.

In addition, the problem requires that households are maximizing intertemporal welfare in (20). The Karush-Kuhn-Tucker conditions for the welfare maximization problem are

$$\left(\frac{cl_{t,r,h}}{n_{t,r,h}}\right)^{-\eta} \geq \lambda_{t,r,h} pcl_{t,r,h} \quad \perp \quad cl_{t,r,h} \geq 0 \tag{116}$$

$$\beta_{t+1,r,h}\lambda_{t+1,r,h} \geq \beta_{t,r,h}\lambda_{t,r,h} \quad \perp \quad kh_{t,r,h} \geq 0, \tag{117}$$

and

$$\begin{aligned} pk_{t+1,r}kh_{t+1,r,h} + pcl_{t,r,h}cl_{t,r,h} \geq & pk_{t,r}kh_{t,r,h} + pl_{t,r}te_{t,r,h} + pr\_ex\_agg_{t,r}kh\_ex_{t,r,h} + \sum_s pres_{t,r,s}resh_{t,r,h,s} \\ & + pfx_t bopdef_{t,r,h} + cpi_t tran_{t,r,h} \quad \perp \quad \lambda_{t,r,h} \geq 0. \end{aligned} \tag{118}$$

Where the level of labor supply is determined by the time constraint, such that

$$te_{t,r,h} \geq leis_{t,r,h} + l_{t,r,h} \quad \perp \quad l_{t,r,h} \geq 0. \tag{119}$$

The problem requires that government's budget constraint holds, as described in Section 2.5, such that

$$\begin{aligned} \sum_r pgov_{t,r}gov_{t,r} + \sum_h tran_{t,r,h} \geq & \sum_r \sum_s ty_{t,r,s}\left(y_{t,r,s} + y\_ex_{t,r,s}\right) + tk_{t,r}\left(kd_{t,r,s} + kd\_ex_{t,r,s} + res_{t,r,s}\right) \\ & + tl_{t,r}ld_{t,r,s} + \sum_h tc_{t,r}cd_{t,r,s,h} \quad \perp \quad incadj_t \geq 0. \end{aligned} \tag{120}$$

To close the finite approximation to the infinite time problem we follow Lau et al. (2002). The capital stock in the post-terminal period, kt_r, is introduced as an endogenous variable with associated price pkt_r. The post-terminal capital stock is determined by requiring that investment is growing at the rate of aggregate consumption growth, such that

$$\frac{inv_{T,r}}{inv_{T-1,r}} \geq \frac{\sum_h c_{T,r,h}}{\sum_h c_{T-1,r,h}} \quad \perp \quad kt \geq 0, \tag{121}$$

where T is the terminal period. The price is determined based on the law of motion for capital, such that

$$k_{T,r}\left(1-\delta\right) + inv_{T,r} \geq kt_r \quad \perp \quad pkt \geq 0. \tag{122}$$

The equations (84)-(122) define the MCP version of the model.

The problem is formulated in the General Algebraic Modeling System (GAMS).[8] The MCP is solved using the PATH solver (Ferris and Munson, 2000). We set the numeraire to the price of foreign exchange, pfx_0 in the intial period.

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